ALTERNATING CURRENT AND ELECTRICAL MACHINES

7.1 Introduction: We have gone through the D.C. current due to batteries/dynamos and the circuit applications of it. We came to known during *electromagnetism* that an emf may be induced / produced in a rotating coil inside a constant uniform magnetic field but the nature of this *induced emf* is *alternating in nature, i.e. it changes its polarity after each half cycle and repeats its waveform over the time period again and again*. The nature of *induced emf*, so produced, is *alternating* and *sinusoidal*. We will discuss the *alternating current* and its generation in this chapter. We will also go through the nature and performance of an *a.c. circuit* containing various elements like *resistors*, *inductors* and *capacitors*.

7.2 Alternating Current: We know from our knowledge of batteries and d.c. current that, *a direct current* (*D.C.*) *has a constant positive value at all the times*, as shown in the Fig. 7.1 (a).

On the other hand, ***the magnitude of an alternating current (A.C.) changes continuously with respect to time and its polarity reverses after every half cycle. The positive half cycle and negative half cycle are mirror images of each other, and the same waveform repeats itself over the time period again and again***. Some of the waveforms for alternating currents are shown in the Fig. 7.1 (b), (c), (d), (e) and (f). The waveforms shown in the Fig. 7.1 (e) and (f) are alternating currents which are varying in sinusoidal manner over the time, and may be produced in a rotating coil rotating at a constant angular speed in a constant uniform magnetic field.

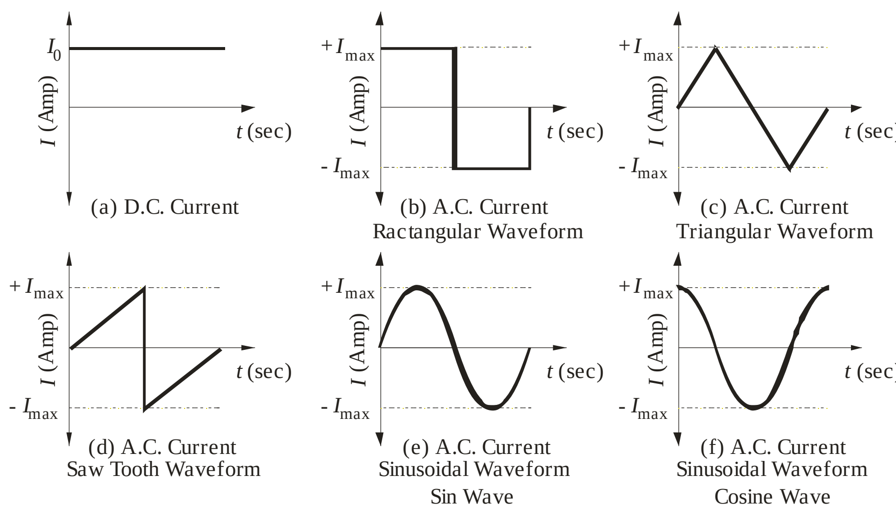


Fig. 7.1

We know from our previous knowledge gathered in ***Electromagnetic Induction*** that, the induced emf in a rotating coil which is rotating at a constant angular speed inside a constant uniform magnetic field may be given as:

$$e = E_0 \sin \omega t \qquad \text{(or, } E_{max} \sin \omega t\text{)} \qquad (7.1)$$

If this emf is applied across a pure resistor of value R, the current flowing through the circuit may be given as:

$$i = \frac{E_0}{R} \sin \omega t = I_0 \sin \omega t \qquad \text{(or, } I_{max} \sin \omega t\text{)} \qquad (7.2)$$

So, the value of maximum / peak current in the circuit may be given as:

$$I_0 = \frac{E_0}{R} \qquad (7.3)$$

7.3 Generation of Single Phase Alternating EMF: We know very well that when a coil is being rotated in a constant uniform magnetic field at a constant angular speed (ω), the change in flux linkages of the coil is sinusoidal and hence a sinusoidal alternating emf is induced in the coil according to the Faraday's law of electromagnetic induction, which may be given by:

$$e = -N\frac{d\phi}{dt} = -N\frac{d}{dt}(\phi_0 \cos \omega t) = N\,\omega\,\phi_0 \sin \omega t$$

or, $e = N\,\omega\,\phi_0 \sin \omega t = e_0 \sin \omega t = e_0 \sin (2\,\pi\,f)\,t$ (7.4)

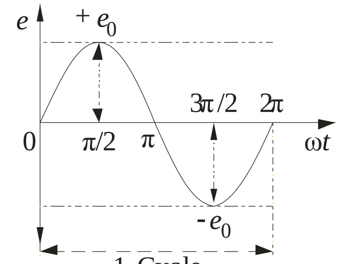


Fig. 7.2 Alternating EMF

(So, induced emf lags the flux by an angle of 90°)

Where, e = instantaneous value of emf induced,

e_0 = maximum value of emf induced,

ω = angular speed (frequency) = $2\,\pi\,f$,

f = supply frequency [frequency of sinusoidal emf / voltage (wave) generated]

Alternating Quantity (Voltage or Current): An *alternating quantity* changes continuously its sign (positive –to– negative and vice–versa) after a fixed time interval and repeats its shortest cycle over the time period again and again.

Waveform (of Voltage or Current): When magnitude of a voltage or current is plotted against the time (or angle of rotating coil) over a graph paper, it is known as the *waveform* of the quantity.

Instantaneous Value: The value of any alternating quantity at a particular instant of time is known as its *instantaneous value*. This *instantaneous value* may be determined from the waveform of the quantity or from the equation of the waveform.

Amplitude (Maximum Value, I_0 or I_{max}): The *maximum value* of the alternating quantity in either direction (positive or negative) is known as *amplitude* of the quantity.

Cycle: The *smallest set of all positive and negative values of an alternating quantity, which repeats itself over the time period again and again,* is known as *one cycle* of the alternating quantity.

Time-period (T): The time taken to complete one cycle of an alternating quantity is known as *time period* (T) of the alternating quantity (wave).

Frequency (f): The number of cycles completed per second by an alternating quantity is known as its *frequency*. It is equal to the *reciprocal of time period.*

Frequency, $f = \dfrac{1}{T}$ Hz (7.5)

Angular Speed (ω) and Supply Frequency (f): Each cycle has one time period (T) and an angular span of 2π radians.

Angular speed, $\omega = \dfrac{2\pi}{T} = 2\,\pi\,f$ rad / sec (7.6)

7.4 Average Value of Alternating Current: It is equal to the value of *D.C.* current, which when flowing through the same element for the same time period transfers the ***same amount of charge***.

$$I_{avg} = \frac{i_1 + i_2 + i_3 + \dots\dots + i_n}{n} = \frac{\sum_{n=1}^{n} i_n}{n} = \frac{1}{n}.\int_0^n (Equation\ of\ \ wave)\, d(variable) \qquad (7.7)$$

The alternating current at any instant of time may be given as:

$i = I_0 \sin \omega t$

We may assume the current constant over very small time period *dt*, the amount of charge transferred during this period may be given as:

$$dq = i\,dt = I_0 \sin \omega t\, dt$$

The total charge that flows through the circuit over a complete cycle of the alternating quantity may be given as:

$$q = \int_0^q dq = \int_0^T I_0 \sin \omega t\, dt \tag{7.8}$$

Now, the average value of the A.C. current flowing through the circuit may be given as:

$$I_{avg} = \frac{q}{T} = \frac{1}{T} \times \int_0^T I_0 \sin \omega t\, dt \tag{7.9}$$

***Average Value* of Alternating Current over Complete Cycle:** The average value of an A.C. current over the complete cycle may be determined as:

$$I_{avg} = \frac{1}{T} \times \int_0^T I_0 \sin \omega t\, dt = \frac{I_0}{T} \times \frac{1}{\omega} \times \left[-\cos \omega t\right]_0^T = \frac{I_0}{T} \times \frac{T}{2\pi} \times \left[\cos \frac{2\pi}{T}\times t\right]_T^0$$

$$= \frac{I_0}{2\pi} \times \left[\cos \frac{2\pi}{T}\times 0 - \cos \frac{2\pi}{T}\times T\right] = \frac{I_0}{2\pi} \times [\cos 0 - \cos 2\pi]$$

or, $$I_{avg} = \frac{I_0}{2\pi} \times [1 - 1] = 0 \tag{7.10}$$

This is not an unusual / unexpected result, as the charge transferred in forward direction during positive half cycle is equal to the charge transferred in backward direction during negative half cycle.

But, the transferring charge in forward direction (in positive half cycle) and then in backward direction (in negative half cycle) is doing some useful work in the electrical circuit during each half cycle. So, the average value of an alternating quantity is to be obtained by doing the average over half cycle.

***Average Value* of Alternating Current over Half Cycle:** The average value of an A.C. current over the half cycle may be determined as:

$$I_{avg} = \frac{1}{(T/2)} \times \int_0^{\frac{T}{2}} I_0 \sin \omega t\, dt = \frac{2I_0}{T} \times \frac{1}{\omega} \times \left[-\cos \omega t\right]_0^{\frac{T}{2}} = \frac{2I_0}{T} \times \frac{T}{2\pi} \times \left[\cos \frac{2\pi}{T}\times t\right]_{\frac{T}{2}}^0$$

$$= \frac{I_0}{\pi} \times \left[\cos \frac{2\pi}{T}\times 0 - \cos \frac{2\pi}{T}\times\frac{T}{2}\right] = \frac{I_0}{\pi} \times [\cos 0 - \cos \pi]$$

or, $$I_{avg} = \frac{I_0}{\pi} \times [1 - (-1)] = \frac{2I_0}{\pi} \tag{7.11}$$

Similarly, $$E_{avg} = \frac{2E_0}{\pi} \tag{7.12}$$

7.5 ***RMS*** **(Root Mean Square) or** ***Virtual*** **or** ***Effective Value*** **of Alternating Current:** It is equal to the value of *D.C.* current, which when flowing through the same element for the same time period produces the ***same amount of heat***.

$$I_{rms} = \sqrt{\frac{i_1^2 + i_2^2 + i_3^2 + \ldots\ldots + i_n^2}{n}} = \sqrt{\frac{1}{n}\int_0^n (Equation\ of\ \ wave)^2\ d(variable)} \qquad (7.13)$$

The alternating current at any instant of time may be given as:

$$i = I_0 \sin \omega t$$

We may assume the current constant over very small time period dt, the small amount of heat produced during this period may be given as:

$$dH = i^2 R\, dt \qquad (7.14)$$

If the time period of the alternating current is T, the heat produced during one cycle may be given as:

$$H = \int_0^H dH = \int_0^T i^2 R\, dt = R \times \int_0^T i^2\ dt \qquad (7.15)$$

If I_{rms} be the effective value of the alternating current, the heat produced by this current in the duration T may be given as:

$$H = I_{rms}^2\ R\,T \qquad (7.16)$$

If heat produced in both the cases is same:

$$H = I_{rms}^2\ R\,T = R \times \int_0^T i^2\ dt$$

or, $$I_{rms}^2 = \frac{1}{T} \times \int_0^T i^2\ dt$$

or, $$I_{rms} = \sqrt{\frac{1}{T} \times \int_0^T i^2\ dt} \qquad (7.17)$$

Now, $$I_{rms} = \sqrt{\frac{1}{T} \times \int_0^T i^2\ dt} = \sqrt{\frac{1}{T} \times \int_0^T (I_0 \sin \omega t)^2\ dt} = \sqrt{\frac{1}{T} \times \int_0^T I_0^2 \sin^2 \omega t\ dt}$$

$$= \sqrt{\frac{I_0^2}{T} \times \int_0^T \left(\frac{1 - \cos 2\omega t}{2}\right) dt} = \sqrt{\frac{I_0^2}{2T} \times \left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T}$$

$$= \sqrt{\frac{I_0^2}{2T} \times \left[\left(T - \frac{\sin 2(2\pi/\mathrm{T}) \times T}{2(2\pi/\mathrm{T})}\right) - \left(0 - \frac{\sin 2(2\pi/\mathrm{T}) \times 0}{2(2\pi/\mathrm{T})}\right)\right]}$$

$$= \sqrt{\frac{I_0^2}{2T} \times \left[\left(T - \frac{\sin 4\pi}{4\pi/\mathrm{T}}\right) - \left(0 - \frac{\sin 0}{4\pi/\mathrm{T}}\right)\right]} = \sqrt{\frac{I_0^2}{2T} \times \left[(T - 0) - (0 - 0)\right]}$$

or, $$I_{rms} = \frac{I_0}{\sqrt{2}} \tag{7.18}$$

7.6 *RMS* (Root Mean Square) or *Virtual* or *Effective Value* of Alternating Emf: It is equal to the value of *D.C.* emf which when applied across the same element for the same time period produces the ***same amount of heat***.

$$E_{rms} = \sqrt{\frac{e_1^2 + e_2^2 + e_3^2 + ... + e_n^2}{n}} = \sqrt{\frac{1}{n}\int_0^n (Equation\ of\ \ wave)^2\ d(variable)} \tag{7.19}$$

The alternating emf at any instant of time may be given as:

$$e = E_0 \sin \omega t$$

We may assume the emf constant over very small time period *dt*, the small amount of heat produced during this period may be given as:

$$dH = \frac{e^2}{R}\ dt \tag{7.20}$$

If the time period of the alternating emf is *T*, the heat produced during one cycle may be given as:

$$H = \int_0^H dH = \int_0^T \frac{e^2}{R}\, dt = \frac{1}{R} \times \int_0^T e^2\, dt \tag{7.21}$$

If E_{rms} be the effective value of the alternating emf, the heat produced by this emf in the duration *T* may be given as:

$$H = \frac{E_{rms}^2}{R} \times T \tag{7.22}$$

If heat produced in both the cases is same:

$$H = \frac{E_{rms}^2}{R} \times T = \frac{1}{R} \times \int_0^T e^2\, dt$$

or, $$E_{rms}^2 = \frac{1}{T} \times \int_0^T e^2\ dt$$

or, $$E_{rms} = \sqrt{\frac{1}{T} \times \int_0^T e^2\ dt} \tag{7.23}$$

So, as similar to the current, we may directly write the *RMS value of alternating emf* as:

$$E_{rms} = \frac{E_0}{\sqrt{2}} \tag{7.24}$$

****:** *All the electrical measuring instruments give the RMS value of any A.C. quantity on their displays.* ***<u>So, always remember that the default given value (without mentioning the name of the value, which value is mentioned) of any A.C. quantity is its RMS value.</u>***

Problem 7.1: *The electric mains in a house is marked as* 220 *V*, 50 *Hz. Write down the equation for instantaneous voltage of the voltage.* [CBSE 1994-95, Haryana 2001-02]

Solution: $E_{rms} = 220$ V, $f = 50$ Hz

The expression for the rms value of alternating emf may be given as:

$$E_{rms} = \frac{E_0}{\sqrt{2}} \qquad \text{or,} \qquad E_0 = \sqrt{2}\ E_{rms} = \sqrt{2} \times 220 = 200\sqrt{2}\ \text{V}$$

Now, the equation for instantaneous voltage / emf may be given as:

$$e = E_0 \sin \omega t = E_0 \sin 2\pi f t = 200\sqrt{2}\ \sin(2\pi \times 50 \times t) = 200\sqrt{2}\ \sin(100\pi t)\ \text{Volts}$$

Problem 7.2: *An electric lamp operates at 12 V d.c. If this lamp is to be connected to an a.c. source and the normal brightness is required, determine the peak value of the a.c. source.*

Solution: $E_{d.c.} = 12$ V

In order to get the normal brightness, the a.c. voltage applied across the lamp must have same rms value as that of the d.c. emf.

So, $$E_{rms} = E_{d.c.} = 12 = \frac{E_0}{\sqrt{2}}$$

or, $$E_0 = \sqrt{2} \times 12 = 16.97\ \text{V}$$

Problem 7.3: *The peak value of an alternating voltage applied to a 50 Ω resistor is 10 V. Determine the rms current, if the voltage frequency is 100 Hz, write down the expression for the instantaneous current.*

Solution: $R = 50\ \Omega$, $E_0 = 10$ V, $f = 100$ Hz

Peak value of the resultant current in the resistor may be given as:

$$I_0 = \frac{E_0}{R} = \frac{10}{50} = 0.2\ \text{A} = 200\ \text{mA}$$

The rms value of the resultant current may be given as:

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{200}{\sqrt{2}} = 100\sqrt{2}\ \text{mA}$$

The expression for the instantaneous value of the current may be given as:

$$i = I_0 \sin \omega t = I_0 \sin(2\pi f t) = 200 \sin(2\pi \times 100 \times t) = 200 \sin(200\pi t)\ \text{m-Amp}$$

Problem 7.4: *Determine the rms value of the current shown in the Fig.* 7.3. [CBSE 1997-98]

Solution: $I_0 = 2$ A

The rms value of the given rectangular waveform may be given as:

$$I_{rms} = \sqrt{\frac{I_1^2 + I_2^2 + I_3^2}{3}} = \sqrt{\frac{(2)^2 + (-2)^2 + (2)^2}{3}} = 2\ \text{A}$$

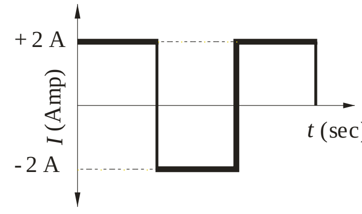


Fig. 7.3

Problem 7.5: *The electric current in a circuit is given as:* $i = i_0\,(t/\tau)$ *at some instant of time. Determine the rms current for the period* $t = 0$ *to* $t = \tau$.

Solution: Given: $i = i_0 \left(\frac{t}{\tau}\right)$

The rms value of the current may be given as:

$$I_{rms} = \sqrt{\frac{1}{\tau} \times \int_0^{\tau} \left(i_0 \times \frac{t}{\tau}\right)^2 dt} = \sqrt{\frac{i_0^2}{\tau^3} \times \int_0^{\tau} t^2\, dt} = \sqrt{\frac{i_0^2}{\tau^3} \times \left[\frac{t^3}{3}\right]_0^{\tau}} = \sqrt{\frac{i_0^2}{\tau^3} \times \left[\frac{\tau^3 - 0^3}{3}\right]} = \frac{i_0}{\sqrt{3}}$$

Problem 7.6: *If the effective value of the current in* 50 *Hz a.c. circuit is* 5 *A. Determine i*) *the peak value of current, ii*) *the mean value of current over a half cycle, iii*) *the value of current after* $\frac{1}{300}$ *sec of its zero value first time.*

Solution: $f = 50$ Hz, $I_{rms} = 5$ A, $t = \frac{1}{300}$ sec

The expression for rms value of the current may be given as:

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

or, $I_0 = \sqrt{2}\ I_{rms} = \sqrt{2} \times 5 = 7.071$ A

The mean value of the current over a half cycle may be given as:

$$I_{avg} = \frac{2 I_0}{\pi} = \frac{2 \times 7.071}{\pi} = 4.502 \text{ A}$$

The instantaneous value of the current at $t = \frac{1}{300}$ sec may be given as:

$$i = I_0 \sin \omega t = I_0 \sin (2\, \pi\, f\, t) = 7.071 \times \sin \left(2\,\pi \times 50 \times \frac{1}{300}\right) = 6.124 \text{ A}$$

*****: Remember that π = 3.14 rad or 180° (put according to unit used).***

Problem 7.7: *The instantaneous value of an alternating voltage across an electrical circuit is given by the expression e* = 140 *sin* 300*t Volts, where t is in seconds. Determine: i*) *the peak value of the voltage, ii*) *the rms value, iii*) *frequency of the supply.*

Solution: $e = 140 \sin 300t$ Volts

Writing the expression of the instantaneous value of alternating emf in standard form:

$$e = 140 \sin \left(2\,\pi \times \frac{300}{2\,\pi} \times t\right) = 140 \sin (2\,\pi \times 47.746 \times t) \text{ Volts}$$

Comparing it with the standard expression: $e = e_0 \sin (2\, \pi\, f\, t)$

$E_0 = 140$ V, $f = 47.746$ Hz

The rms value of the voltage may be given as:

$$E_{rms} = \frac{E_0}{\sqrt{2}} = \frac{140}{\sqrt{2}} = 98.995 \text{ V}$$

Problem 7.8: *A resistance of* 40 Ω *is connected to an a.c. source of* 220 *V,* 50 *Hz. Determine: i) the rms current, ii) the maximum instantaneous current in the resistor, iii) the time taken by the current to change from its maximum value to the rms value.*

Solution: $R = 40\ \Omega$, $V_{rms} = 220$ V, $f = 50$ Hz

The rms value of the current through the resistor may be given as:

$$I_{rms} = \frac{V_{rms}}{R} = \frac{220}{40} = 5.5 \text{ A}$$

The maximum value of the instantaneous current may be given as:

$$I_0 = \sqrt{2} \times I_{rms} = \sqrt{2} \times 5.5 = 7.778 \text{ A}$$

The expression for the instantaneous current may be written as:

$$i = I_0 \sin \omega t = 7.778 \sin (2\,\pi \times 50 \times t) = 7.778 \sin (100\,\pi\, t) \text{ Amp}$$

Since, we have to determine the time taken by the current to change from its maximum value to the rms value, shifting the origin to $\frac{\pi}{2}$ radian. So, the expression of the current becomes:

$$i = 7.778 \sin (100\,\pi\, t + \pi/2) = 7.778 \cos (100\,\pi\, t) \text{ Amp}$$

The time taken may be calculated as:

$$I_{rms} = 7.778 \cos (100\,\pi\, t) = 5.5$$

or, $$t = \frac{1}{100\pi} \times \cos^{-1}\left(\frac{5.5}{7.778}\right) = 2.5 \times 10^{-3} \text{ sec} = 2.5 \text{ m-sec}$$

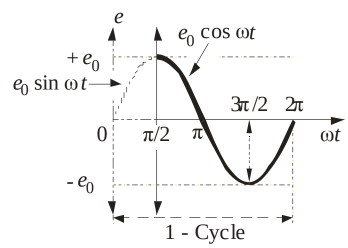


Fig. 7.4

*****: Remember that π = 3.14 rad or 180° (put according to unit used).***

Problem 7.9: *The instantaneous emf of an a.c. source is given as: e =* 300 *sin* 314*t Volts. Determine the rms value of the emf.* [CBSE 1995-96, 1999-2000]

Solution: $e = 300 \sin 314t$ Volts

So, $E_0 = 300$ V

The rms value of the emf may be given as:

$$E_{rms} = \frac{E_0}{\sqrt{2}} = \frac{300}{\sqrt{2}} = 212.132 \text{ V}$$

Problem 7.10: *The instantaneous emf of an a.c. source is given as: e =* 300 *sin* 314*t Volts. Determine the peak value of the emf and frequency of the source.* [CBSE 1992-93]

Solution: $e = 300 \sin 314t$ Volts

Writing the expression of the instantaneous value of alternating emf in standard form:

$$e = 300 \sin\left(2\pi \times \frac{314}{2\pi} \times t\right) = 300 \sin (2\,\pi \times 50 \times t) \text{ Volts}$$

Comparing it with the standard expression: $e = e_0 \sin (2\,\pi\, f\, t)$

$E_0 = 300$ V, $f = 50$ Hz

Problem 7.11: *The instantaneous current from an a.c. source is given as:* $i = 5 \sin 314t$ *Amp. Determine the rms value of current and frequency of the source.* [CBSE PMT 1999-2000]

Solution: $i = 5 \sin 314t$ Amp

Writing the expression of the instantaneous value of alternating current in standard form:

$$i = 5 \sin\left(2\pi \times \frac{314}{2\pi} \times t\right) = 5 \sin (2\,\pi \times 50 \times t) \text{ Amp}$$

Comparing it with the standard expression: $i = I_0 \sin (2\,\pi\, f\, t)$

$I_0 = 5$ A, $f = 50$ Hz

Now, the rms value of the current may be given as:

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{5}{\sqrt{2}} = 3.536 \text{ A}$$

Problem 7.12: *An alternating current is given as:* $i = 50 \sin (400\,\pi t + \phi)$ *Amp. Determine the frequency and the rms value of the current.*

Solution: $i = 50 \sin (400\,\pi t + \phi)$ Amp

Writing the expression of the instantaneous value of alternating current in standard form:

$$i = 50 \sin\left(2\pi \times \frac{400\pi}{2\pi} \times t + \phi\right) = 50 \sin (2\,\pi \times 200 \times t) \text{ Amp}$$

Comparing it with the standard expression: $i = I_0 \sin (2\,\pi\, f\, t)$

$I_0 = 50$ A, $f = 200$ Hz

Now, the rms value of the current may be given as:

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{50}{\sqrt{2}} = 25\sqrt{2} \text{ A} = 35.36 \text{ A}$$

Problem 7.13: *An alternating emf of peak value* 350 *V is applied across a resistor of* 100 Ω *through an a.c. ammeter. Determine the reading of the ammeter.*

Solution: $E_0 = 350$ V, $R = 100\ \Omega$

The peak value of the current through the ammeter and the resistor may be given as:

$$I_0 = \frac{E_0}{R} = \frac{350}{100} = 3.5 \text{ A}$$

Since, all the measuring instruments show the rms value on their displays. So, the reading of the ammeter may be given as:

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{3.5}{\sqrt{2}} = 2.475 \text{ A}$$

Problem 7.14: *The effective value of the current in a* 50 *Hz a.c. circuit is* 5 *A. Determine the value of current after* $\frac{1}{300}$ *sec from its zero value and increasing in positive direction.* [Punjab 1993-94]

Solution: $f = 50$ Hz, $I_{rms} = 5$ A, $t = \frac{1}{300}$ sec (from zero value)

The instantaneous value of the current may be given as:

$$i = I_0 \sin \omega t = \sqrt{2}\ I_{rms} \sin (2\ \pi\ f\ t) = \sqrt{2}\ \times 5 \times \sin\left(2\pi \times 50 \times \frac{1}{300}\right) = 6.124 \text{ A}$$

Problem 7.15: *The peak value of an alternating current of frequency* 50 *Hz is* 14.14 *A. Determine its rms value. How much time will the current take in reaching from* 0 *A to its maximum value?*

Solution: $f = 50$ Hz, $I_0 = 14.14$ A

The rms value of the current may be given as:

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{14.14}{\sqrt{2}} = 10 \text{ A}$$

The time taken by the current in reaching from 0 A to its maximum value is quarter of its time period.

So, $$t = \frac{T}{4} = \frac{1}{4 \times f} = \frac{1}{4 \times 50} = 5 \times 10^{-3} \text{ sec} = 5 \text{ m-sec}$$

Alternatively:

$$i = 14.14 \sin (2\ \pi \times 50 \times t) = 14.14$$

or, $$t = \frac{1}{100\pi} \times \sin^{-1}\left(\frac{14.14}{14.14}\right) = 5 \times 10^{-3} \text{ sec} = 5 \text{ m-sec}$$

Problem 7.16: *A* 100 Ω *iron is connected to a* 220 *V,* 50 *Hz supply. Determine: i*) *peak potential difference across the iron, ii*) *average potential difference across the iron, iii*) *rms value of current drawn by the iron.*

Solution: $R = 100\ \Omega$, $V_{rms} = 220$ V, $f = 50$ Hz

The peak value of potential difference across the iron may be given as:

$$V_0 = \sqrt{2} \times V_{rms} = \sqrt{2} \times 220 = 311.127 \text{ V}$$

The average potential difference across the iron may be given as:

$$V_{avg} = \frac{2V_0}{\pi} = \frac{2 \times 311.127}{\pi} = 198.07 \text{ V}$$

The rms value of the current drawn by the iron may be given as:

$$I_{rms} = \frac{V_{rms}}{R} = \frac{220}{100} = 2.2 \text{ A}$$

Problem 7.17: *The expression for a.c. current flowing through an electrical circuit is i* = 50 *sin* 100*πt Amp. Determine: i*) *frequency of the supply, ii*) *mean value of the a.c. current over positive half cycle, iii*) *rms value of the current, iv*) *the value of current after* $\frac{1}{300}$ *sec from the instant when it was zero ampere.*

Solution: $i = 50 \sin (100\,\pi\, t)$ Amp

Writing the expression of the instantaneous value of alternating current in standard form:

$$i = 50 \sin \left(2\pi \times \frac{100\pi}{2\pi} \times t\right) = 5 \sin (2\,\pi \times 50 \times t) \text{ Amp}$$

Comparing it with the standard expression: $i = I_0 \sin (2\,\pi\, f\, t)$

$I_0 = 50$ A, $\quad f = 50$ Hz

The mean value of the current over positive half cycle may be given as:

$$I_{avg} = \frac{2 I_0}{\pi} = \frac{2 \times 50}{\pi} = 31.83 \text{ A}$$

The rms value of the current may be given as:

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{50}{\sqrt{2}} = 35.36 \text{ A}$$

The instantaneous value of the current may be given as:

$$i = I_0 \sin \omega t = I_0 \sin (2\,\pi\, f\, t) = 50 \times \sin\left(2\pi \times 50 \times \frac{1}{300}\right) = 43.3 \text{ A}$$

Problem 7.18: *An alternating current of frequency* 60 *Hz has a maximum value of* 120 *A. Write down the equation for its instantaneous value. Find the time taken to reach* 96 *A for the first time.*

Solution: $f = 60$ Hz, $\quad I_0 = 120$ A

The expression for the instantaneous value of current may be given as:

$$i = I_0 \sin 2\pi f\, t = 120 \sin (2\,\pi \times 60\, t) = 120 \sin (120\,\pi\, t) \text{ Amp}$$

Now for time taken:

$$i = 120 \sin (120\,\pi\, t) = 96$$

or, $$120\,\pi\, t = \sin^{-1}\left(\frac{96}{120}\right) = 53.13° = 0.927 \text{ radians}$$

or, $$t = \frac{53.13°}{120 \times 180°} = \frac{0.927}{120 \times (22/7)} = 2.46 \text{ milli-sec}$$

7.7 Phasor Diagram of Sinusoidal *A.C.* Quantities: Any sinusoidal *A.C.* quantity is equivalent to a rotating vector at a constant angular speed (ω) and its instantaneous value is equal to the projection of that rotating vector at *y*-axis at any instant of time (as shown in Fig. 7.5 just like the true *S.H.M.*).

So, ***they may be represented by stationary vectors on a diagram known as phasor diagram, if and only if all the vectors are rotating at the same constant angular speed* (ω)*, so that they are stationary w.r.t. each other. This is the reason of standardizing the frequency of supply in India at* 50 *Hz and at* 50 / 60 *Hz in other countries.***

All phasor diagrams for practical purposes are drawn using RMS (effective) values of *A.C.* quantities.

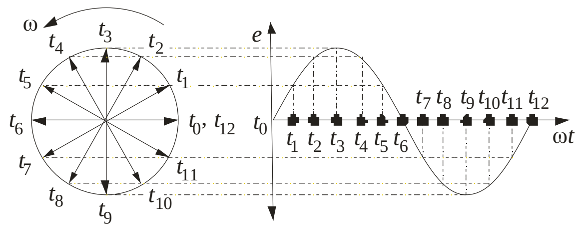


Fig. 7.5

Phase: Phase of an ac quantity means "*the time period which has been elapsed after the origin when the a.c. quantity passes through the zero value first time after the origin and increasing in positive direction there-afterwards*". [Fig. 7.6(a)]

Phase Angle: "*Angle between the phasor of an a.c. quantity and the reference line is known as phase angle*" of that *a.c.* quantity [Fig. 7.6(b)]. So, phase and phase angle of an *a.c.* quantity are same.

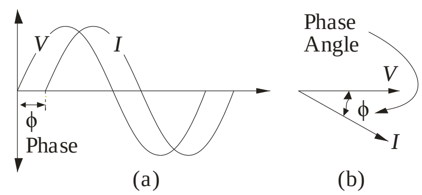


Fig. 7.6

Phase Difference: Angle between two *A.C.* quantities is known as the phase difference between these *A.C.* quantities.

e.g. If, $v = V_0 \sin \omega t$, and $i_1 = I_{01} \sin (\omega t - 60°)$, and $i_2 = I_{02} \sin (\omega t - 100°)$

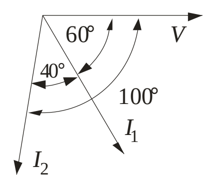


Fig. 7.7

Refer to Fig. 7.7:

Phase of $v = 0°$, Phase of $i_1 = -60°$, Phase of $i_2 = -100°$

Phase difference between v and $i_1 = 0° - (-60°) = 60°$ (i_1 lagging behind v)

and, Phase difference between i_1 and $i_2 = -60° - (-100°) = 40°$ (i_2 lagging behind i_1)

7.8 Purely Resistive Circuit: In a purely resistive circuit applied voltage and the current in circuit are co-phasor (in same phase, refer to Fig. 7.8) and as explained below:

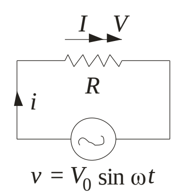


Fig. 7.8

Applied voltage is, $v = V_0 \sin \omega t$ (7.25)

Now, $v = V_0 \sin \omega t = i\,R$

or, $$i = \frac{V_0}{R} \sin \omega t = I_0 \sin \omega t \qquad (7.26)$$

****: Note that the phase angle of *voltage* as well as *current* wave is zero, as shown in the Fig. 7.9 (a) and (b). So, the phase difference between voltage and current wave is also zero, *i.e. voltage and current are co-phasor*.**

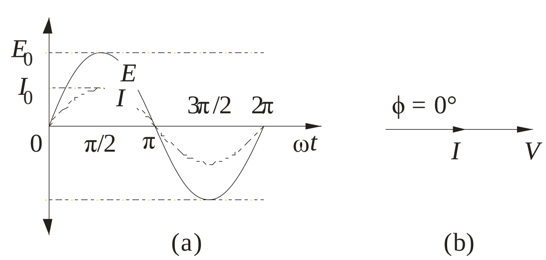


Fig. 7.9

Examining equations (7.25) and (7.26) and the Phasor Diagram in Fig. 7.9 (b): The reader may easily conclude that:

$$I_0 = \frac{V_0}{R} \text{ and angle between } v \text{ and } i = 0° \qquad (7.27)$$

Power Consumed in a Purely Resistive Circuit: The instantaneous power in the circuit may be given by the product of instantaneous voltage and instantaneous current:

$$p = v\,i = V_0 \sin \omega t \times I_0 \sin \omega t = V_0\, I_0 \sin^2 \omega t = V_0\, I_0 \left(\frac{1-\cos 2\omega t}{2}\right)$$

Now, $$P_{avg} = \frac{1}{\pi} \times \int_0^{\pi} V_0\, I_0 \left(\frac{1-\cos 2\omega t}{2}\right) d(\omega t) = \frac{V_0\, I_0}{2\pi} \times \left[\omega t - \frac{\sin 2\omega t}{2}\right]_0^{\pi}$$

$$= \frac{V_0\, I_0}{2\pi} \times [(\pi - 0) - (0 - 0)] = \frac{V_0\, I_0}{2} = \frac{V_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}} = V_{rms}\, I_{rms}$$

or, $$P_{avg} = V\, I \text{ Watts} \qquad (7.28)$$

7.9 Purely Inductive Circuit: When an alternating current flows through a coil (*pure inductor*), the coil is associated with the alternating flux produced due to the alternating current flowing through itself. So, in a purely inductive circuit an emf is being induced in the coil (*pure inductor*), which is directly proportional to the rate of change of flux and hence the rate of change of current flowing through itself.

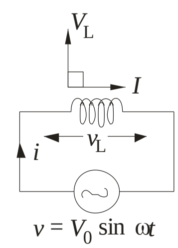


Fig. 7.10

Applied voltage and emf induced in the inductor are same (refer to Fig. 7.10).

$$v = V_0 \sin \omega t = v_L \propto \frac{d\phi}{dt} \propto \frac{di}{dt}$$

or, $$V_0 \sin \omega t = v_L = L \times \frac{di}{dt} \qquad (7.29)$$

(Where, *L is proportionality constant and is known as* ***self inductance of the coil / inductor***).

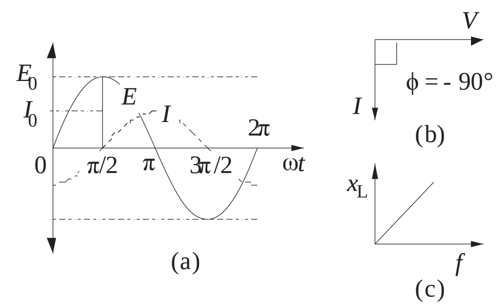


Fig. 7.11

Now, $$i = \int \frac{1}{L} \times v_L\, dt$$

$$= \frac{1}{L} \times \int V_0 \sin \omega t\, dt = -\frac{V_0}{\omega L} \cos \omega t$$

or, $$i = \frac{V_0}{\omega L} \times \sin\left(\omega t - \frac{\pi}{2}\right) = \frac{V_0}{X_L} \times \sin\left(\omega t - \frac{\pi}{2}\right) = I_0 \sin\left(\omega t - \frac{\pi}{2}\right) \qquad (7.30)$$

Examining equation (7.29) and (7.30) and the Phasor Diagram in Fig. 7.11 (b): The reader may easily conclude that:

$$I_0 = \frac{V_0}{\omega L} = \frac{V_0}{X_L} \quad \text{and} \quad \phi = \frac{\pi}{2} = 90° \text{ (lagging)} \qquad (7.31)$$

$X_L = \omega L = 2\pi f L$; is the ***inductive reactance*** (***reactance offered by the inductor to flow of current***).

Also, ***current*** **lags behind the *voltage* by 90°**. Variation of the inductive reactance (X_L) *w.r.t.* the supply frequency is also shown in the Fig. 7.11 (c).

Note that: If the power supply across the inductor is d.c., *i.e.* the supply frequency $f = 0$ then, the inductive reactance $\boldsymbol{X_L = \omega L = 2\pi f L = 0}$. So, ***an inductor behaves as short circuit for the d.c. supply at its steady state operation***.

Power Consumed in a Purely Inductive Circuit: The instantaneous power in the circuit may be given by the product of instantaneous voltage and instantaneous current:

$$p = v\,i = V_0 \sin \omega t \times I_0 \cos \omega t = \frac{V_0 I_0}{2} \sin 2\omega t$$

Now, $$P_{avg} = \frac{1}{\pi} \times \int_0^{\pi} \left[\frac{V_0 I_0}{2} \sin 2\omega t \right] d(\omega t) = \frac{V_0 I_0}{2\pi} \times \left[-\frac{\cos 2\omega t}{2} \right]_0^{\pi}$$

$$= \frac{V_0 I_0}{4\pi} \times [\cos 2\omega t]_{\pi}^{0} = \frac{V_0 I_0}{4\pi} \times [\cos 0 - \cos \pi] = \frac{V_0 I_0}{4\pi} \times [1 - 1] = 0 \text{ Watts}$$

So, $$P_{avg} = 0 \text{ Watts} \tag{7.32}$$

Hence, ***no power is being dissipated in a purely inductive circuit***. Although, inductive power is there in the circuit, oscillating at the double of the supply frequency (2ω), but ***average power dissipated is zero***. So, *an inductor is an energy storing device, which stores the power in half cycle and returns it back to the circuit in next half cycle*.

The current in the circuit is lagging behind the supply voltage by 90°, and the power dissipated in the circuit due to this current is zero. So, this current is known as ***wattles current / virtual current***.

7.10 Purely Capacitive Circuit: In a purely capacitive circuit opposite charges are being stored on the plates of the capacitor due to the flow of an alternating current, the applied alternating voltage and the charge stored on the capacitor plates may respectively be given as: (refer to Fig. 7.12).

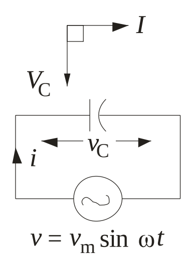


Fig. 7.12

$$v_C = V_0 \sin \omega t \tag{7.33}$$

and, $$q = C\,v_C$$

and, $$i = \frac{dq}{dt} = \frac{d}{dt}(C\,V_0 \sin \omega t) = \omega\,C\,V_0 \cos \omega t$$

or, $$i = \frac{V_0}{(1/\omega C)} \times \sin\left(\omega t + \frac{\pi}{2}\right) = \frac{V_0}{X_C} \times \sin\left(\omega t + \frac{\pi}{2}\right)$$

or, $$i = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) \tag{7.34}$$

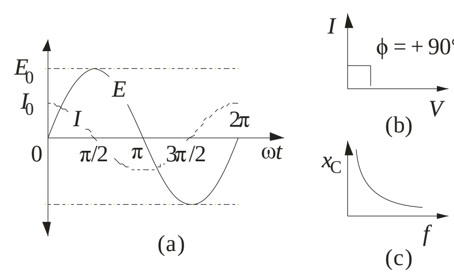


Fig. 7.13

Examining equation (7.33) and (7.34) and the Phasor Diagram in Fig. 7.13 (b): The reader may easily conclude that:

$$I_0 = \frac{V_0}{(1/\omega C)} = \frac{V_0}{X_C} \tag{7.35.1}$$

and, $\phi = \frac{\pi}{2} = 90°$ (leading) (7.35.2)

$\boldsymbol{X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}}$; is ***capacitive reactance*** (***reactance offered by the capacitor to flow of current***).

Also, ***current* leads the *voltage* by 90°**. Variation of the capacitive reactance (X_C) *w.r.t.* the supply frequency is shown in Fig. 7.13 (c).

Note that: If the power supply across the capacitor is d.c., *i.e.* the supply frequency $f = 0$ then, the capacitive reactance $\boldsymbol{X_C} = \frac{1}{\boldsymbol{\omega C}} = \frac{1}{\boldsymbol{2\pi f C}} = \infty$**.** So, ***a capacitor behaves as open circuit for the d.c. supply at its steady state operation***.

Power Consumed in a Purely Capacitive Circuit: The instantaneous power in the circuit may be given by the product of instantaneous voltage and instantaneous current:

$$p = v\,i = V_0 \sin \omega t \times I_0 \cos \omega t = \frac{V_0 I_0}{2} \sin 2\omega t$$

Now,
$$P_{avg} = \frac{1}{\pi} \times \int_0^{\pi} \left[\frac{V_0 I_0}{2} \sin 2\omega t \right] d(\omega t) = \frac{V_0 I_0}{2\pi} \times \left[-\frac{\cos 2\omega t}{2} \right]_0^{\pi}$$

$$= \frac{V_0 I_0}{4\pi} \times [\cos 2\omega t]_{\pi}^{0} = \frac{V_0 I_0}{4\pi} \times [\cos 0 - \cos \pi] = \frac{V_0 I_0}{4\pi} \times [1 - 1] = 0 \text{ Watts}$$

So, $P_{avg} = 0$ Watts (7.36)

Hence, ***no power is being dissipated in a purely capacitive circuit***, as similar to a purely inductive circuit. Although, capacitive power is there in the circuit, oscillating at the double of the supply frequency (2ω), but ***average power dissipated is zero***. So, *a capacitor is also an energy storing device, which stores the power in half cycle and returns it back to the circuit in next half cycle*.

The current in the circuit is leading the supply voltage by 90°, and the power dissipated in the circuit due to this current is zero. So, this current is known as ***wattles current* / *virtual current***.

Effect of a Capacitor in D.C. Circuit: When a capacitor is connected across a d.c. supply source, it starts to get charged due to unidirectional current flowing through the capacitor due to the d.c. supply source.

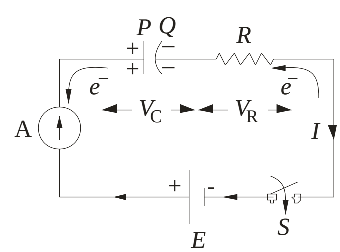


Fig. 7.14

Consider the capacitor connected across a d.c. battery through a resistor (R), a tapping switch (S) and an ammeter (A), as shown in the Fig. 7.14. As soon as the switch S is closed, the electrons from the plate P of the capacitor starts to flow through the circuit towards the plate Q of the capacitor under the influence of electric field due to the emf of the battery. So, the plate P of the capacitor gets to acquire a positive charge, while the plate Q of the capacitor gets to acquire an equal negative charge. The capacitor starts charging in this way and a new electric field, due to the positive charge on plate P and the negative charge on plate Q, builds up in the opposite direction of the electric field due to battery. So, the current in the circuit decreases with elapsing time, and finally becomes zero, when the electric field due to charge on capacitor plates becomes equal to the electric field due to emf of the battery. It again proves that a capacitor behaves as open circuit for a d.c. supply source at its steady state operation.

The individual voltages across the two elements (resistor and capacitor) may respectively be given as:

$$V_R = i\,R \tag{7.37}$$

and, $$V_C = \frac{q}{C} \tag{7.38}$$

Now, applying Kirchhoff's Voltage Law in the circuit:

$$E = V_R + V_C = i\,R + \frac{q}{C} = R\,\frac{d\,q}{d\,t} + \frac{q}{C}$$

or, $$\frac{C\,E - q}{C} = R\,\frac{d\,q}{d\,t}$$

or, $$d\,t = \frac{R\,C}{C\,E - q}\,d\,q \tag{7.39}$$

Integrating above equation:

$$t = -\,R\,C \ln (C\,E - q) + A \tag{7.40}$$

Now at the time t = 0, the charge deposited on the capacitor is zero, *i.e.* q = 0. Putting these values in equation (7.40):

$$0 = -\,R\,C \ln (C\,E - 0) + A$$

or, $$A = R\,C \ln (C\,E) \tag{7.41}$$

So, $$t = -\,R\,C \ln (C\,E - q) + R\,C \ln (C\,E) = -\,R\,C\,[\ln (C\,E - q) - \ln (C\,E)]$$

or, $$-\,\frac{t}{RC} = \ln \left(\frac{C\,E - q}{C\,E}\right)$$

or, $$e^{-(t/RC)} = \frac{C\,E - q}{C\,E} = 1 - \frac{q}{C\,E}$$

So, $$q = C\,E\,(1 - e^{-(t/RC)}) \tag{7.42}$$

The current through the circuit may be given as:

$$i = \frac{d\,q}{d\,t} = \frac{d}{d\,t}\,[C\,E\,(1 - e^{-(t/RC)})] = C\,E\left[0 - \left(-\frac{1}{RC}\right) \times e^{-(t/RC)}\right]$$

or, $$i = \frac{E}{R} \times e^{-(t/RC)} = \frac{E}{R} \times e^{-(t/\tau)} \tag{7.43}$$

Where, τ = time constant of the (RC) circuit = RC

At a time $t = \tau$:

$$q = C\,E\,(1 - e^{-(t/RC)}) = C\,E\,(1 - e^{-(\tau/\tau)}) = C\,E\,(1 - e^{-1}) = C\,E\left(1 - \frac{1}{e}\right) = C\,E\left(\frac{e - 1}{e}\right)$$

or, $$q = 0.632 \times C\,E = 0.632 \times q_0 = 0.632 \times \text{steady state charge on the capacitor}$$

SO, ***"the time constant may be defined as the time taken by the capacitor to charge* 0.632 *times the steady state charge on the capacitor"***.

At a sufficiently large time, *i.e.* at $t = \infty$.

$$i\,(t) = \frac{E}{R} \times e^{-(\infty/RC)} = \frac{E}{R} \times \frac{1}{e^{(\infty/RC)}} = \frac{E}{R} \times \frac{1}{\infty} = 0$$

i.e. the capacitor behaves as open circuit after getting fully charged to the source voltage and hence current becomes zero in the circuit at steady state.

Problem 7.19: *A* 100 *Hz a.c. supply source is applied across a* 14 *mH coil. Determine the reactance of the coil offered to the flow of current.* [Haryana 1997-98]

Solution: $f = 100$ Hz, $L = 14$ mH

The reactance offered by the inductor to flow of current may be given as:

$$X_L = \omega\,L = 2\,\pi\,f\,L = 2\,\pi \times 100 \times 14 \times 10^{-3} = 8.797\ \Omega$$

Problem 7.20: *A pure inductor of* 25 *mH is connected to a source of* 220 *V,* 50 *Hz. Determine the inductive reactance and rms current flowing through the circuit.* [NCERT]

Solution: $L = 25$ mH, $V = 220$ V, $f = 50$ Hz

The reactance offered by the inductor to flow of current may be given as:

$$X_L = \omega\,L = 2\,\pi\,f\,L = 2\,\pi \times 50 \times 25 \times 10^{-3} = 7.854\ \Omega$$

So, the rms value of the current flowing through the circuit may be given as:

$$I = \frac{V_{rms}}{X_L} = \frac{220}{7.854} = 28.011\ \text{A}$$

Problem 7.21: *Determine the maximum value of current when an inductance of* 1 *H is connected to an a.c. source of* 200 *V,* 50 *Hz.* [CBSE 1994-95, Punjab 1999-2000]

Solution: $L = 1$ H, $V = 200$ V, $f = 50$ Hz

The maximum value of the current flowing through the inductor may be given as:

$$I_0 = \sqrt{2} \times I_{rms} = \sqrt{2} \times \frac{V_{rms}}{X_L} = \sqrt{2} \times \frac{V_{rms}}{2\pi\,f\,L} = \sqrt{2} \times \frac{200}{2\pi \times 50 \times 1} = 0.9\ \text{A}$$

Problem 7.22: *A coil has an inductance of* 1 *H. Determine: i*) *the required frequency, when it offers an inductive reactance of* 3142 Ω *to the flow of current, ii*) *the value of capacitance, which has the same reactance at this frequency.* [CBSE 1994-95]

Solution: $L = 1$ H, $X_L = 3142\ \Omega$

The expression for inductive reactance offered by an inductor to the flow of current may be given as:

$$X_L = 2\,\pi\,f\,L$$

or, $$f = \frac{X_L}{2\pi L} = \frac{3142}{2 \times \pi \times 1} = 500\ \text{Hz}$$

The expression for capacitive reactance offered by a capacitor to the flow of current may be given as:

$$X_C = \frac{1}{2\,\pi\,f\,C}$$

or, $$C = \frac{1}{2\pi\, f\, X_C} = \frac{1}{2\,\pi \times 500 \times 3142} = 101.31 \times 10^{-9}\text{ F} = 101.31\text{ nF}$$

Problem 7.23: *An a.c. circuit consists of only an inductor of inductance* 2 *H. If the current through it is represented by a sine wave of amplitude* 0.25 *A with a frequency of* 60 *Hz, determine the effective potential difference across the inductor.*

Solution: L = 2 H, I_0 = 0.25 A, f = 60 Hz

The effective (rms) value of potential difference across the inductor may be given as:

$$V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{I_0\, X_L}{\sqrt{2}} = \frac{I_0 \times (2\pi\, f\, L)}{\sqrt{2}} = \frac{0.25 \times (2\,\pi \times 60 \times 2)}{\sqrt{2}} = 133.286\text{ V}$$

Problem 7.24: *An alternating emf* e = 220 *sin* (100 π *t*) *Volts is applied across an inductor of* $\frac{1}{\pi}$ *H. Write down an equation for instantaneous current through the circuit. Also, determine the reading of an a.c. ammeter, if connected in the circuit.*

Solution: e = 220 sin (100 π t) Volts, $L = \frac{1}{\pi}$ H

Comparing above equation with: $e = E_0 \sin \omega t$

E_0 = 220 V, ω = 100 π rad/sec

The instantaneous current through the circuit may be given as:

$$i = \frac{e}{X_L} = \frac{e}{\omega L} = \frac{220}{100\,\pi \times (1/\pi)}\ \sin(100\,\pi\, t) = 2.2 \sin(100\,\pi\, t)\text{ Amp}$$

Since, all the measuring instruments give the rms value on their displays, so the reading of the ammeter may be given as:

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{2.2}{\sqrt{2}} = 1.556\text{ A}$$

Problem 7.25: *An inductor of inductance* 200 *mH is connected to an a.c. source of peak emf* 210 *V,* 50 *Hz. Determine the peak current through the inductor. Also, determine the value of instantaneous voltage across the inductor at the instant when the instantaneous current is at its peak value.*

Solution: L = 200 mH, V_0 = 210 V, f = 50 Hz

The value of the peak current through the inductor may be given as:

$$I_0 = \frac{V_0}{X_L} = \frac{V_0}{2\pi\, f\, L} = \frac{210}{2\,\pi \times 50 \times 0.200} = 3.342\text{ A}$$

Since the current through the purely inductive circuit lags behind the supply voltage by 90°, so the value of instantaneous voltage at the instant when the instantaneous current is at its peak value may be given as:

$$v_{(\text{when } i \text{ is at its peak value})} = 0\text{ Volt}$$

Problem 7.26: *A* 15 μ*F capacitor is connected across a* 220 *V,* 50 *Hz source. Determine the capacitive reactance and the current* (*rms and peak value*) *flowing through the circuit. If the frequency of the supply source is doubled, what happens to the capacitive reactance and the current?* [NCERT]

Solution: C = 15 μF, V = 220 V, f = 50 Hz

The capacitive reactance offered by a capacitor to the flow of current may be given as:

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 15 \times 10^{-6}} = 212.21\ \Omega$$

The rms value of the current flowing through the circuit may be given as:

$$I_{rms} = \frac{V_{rms}}{X_C} = \frac{220}{212.21} = 1.037\ \text{A}$$

The peak value of the current flowing through the circuit may be given as:

$$I_0 = \sqrt{2} \times I_{rms} = \sqrt{2} \times 1.037 = 1.467\ \text{A}$$

If the frequency got doubled, the capacitive reactance (being $X_C \propto \frac{1}{f}$) becomes halved of the earlier value, and the current (being $I \propto \frac{1}{X_C} \propto f$) becomes double of the earlier value.

So, $$X'_C = \frac{X_C}{2} = \frac{212.21}{2} = 106.105\ \Omega$$

and, $$I'_{rms} = 2 \times I_{\text{rms}} = 2 \times 1.037 = 2.074\ \text{A}$$

and, $$I'_0 = 2 \times I_0 = 2 \times 1.467 = 2.934\ \text{A}$$

Problem 7.27: *A* 1 μ*F capacitor is connected across a supply source having emf* $e = 250 \sin(100\,\pi\, t)$ *Volts. Write down the expression for the instantaneous current flowing through the circuit and determine the reading of an a.c. ammeter connected in the circuit.*

Solution: $C = 1\ \mu\text{F}$, $e = 250 \sin(100\,\pi\, t)$ Volts

Comparing above equation with: $e = E_0 \sin \omega t$

$$E_0 = 250\ \text{V}, \qquad \omega = 100\,\pi\ \text{rad/sec}$$

The instantaneous current through the circuit may be given as:

$$i = \frac{e}{X_C} = \omega\, C\, e = 100\,\pi \times 1 \times 10^{-6} \times 250 \sin(100\,\pi\, t)$$

or, $$i = 0.0785 \sin(100\,\pi\, t)\ \text{Amp} = 78.5 \sin(100\,\pi\, t)\ \text{m-Amp}$$

Since, all the measuring instruments give the rms value on their displays, so the reading of the ammeter may be given as:

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{78.5}{\sqrt{2}} = 55.51\ \text{mA}$$

Problem 7.28: *Determine the inductive reactance of a coil, if current flowing through it is 800 mA when an a.c. source of* 40 *V is applied across it.* [Haryana 1990-91]

Solution: $I = 800$ mA, $V = 40$ V

The expression for the voltage across an inductor may be given as:

$$V = I\, X_L$$

or, $$X_L = \frac{V}{I} = \frac{40}{0.800} = 50\ \Omega$$

Problem 7.29: *Determine the value of current flowing through an inductor of* 2 *H and negligible resistance, when connected across an a.c. source of* 150 *V,* 50 *Hz.* [Punjab 1991-92]

Solution: $L = 2$ H, $V = 150$ V, $f = 50$ Hz

The current flowing through an inductor may be given as:

$$I = \frac{V}{X_L} = \frac{V}{2\pi f L} = \frac{150}{2\pi \times 50 \times 2} = 0.239 \text{ A}$$

Problem 7.30: *An inductor having an inductive reactance of* 22 Ω *at* 200 *Hz and a negligible resistance is connected across* 220 *V,* 50 *Hz a.c. supply source. Determine the value of inductance and reactance at this new frequency.*

Solution: $X_{L1} = 22\ \Omega$ (at $f_1 = 200$ Hz), $V = 220$ V, $f_2 = 50$ Hz

The expression for the inductive reactance may be given as:

$$X_{L1} = 2\pi f_1 L$$

or, $$L = \frac{X_{L1}}{2\pi f_1} = \frac{22}{2\pi \times 200} = 17.5 \text{ mH (inductance is independent of frequency)}$$

The ratio inductive reactance at two different frequencies may be given as:

$$\frac{X_{L2}}{X_{L1}} = \frac{2\pi f_1 L}{2\pi f_2 L} = \frac{f_1}{f_2}$$

So, $$X_{L2} = \frac{f_1}{f_2} \times X_{L1} = \frac{50}{200} \times 22 = 5.5\ \Omega$$

Problem 7.31: *An inductive coil has inductive reactance of* 88 Ω *at* 50 *Hz. Determine the self inductance of the coil.*

Solution: $X_L = 88\ \Omega$ (at $f = 50$ Hz)

The expression for the inductive reactance may be given as:

$$X_L = 2\pi f L$$

or, $$L = \frac{X_L}{2\pi f} = \frac{88}{2\pi \times 50} = 280.112 \text{ mH}$$

Problem 7.32: *Determine the maximum current through an inductor of* 2 *H connected across an a.c. source of* 150 *V,* 50 *Hz.* [Punjab 1996-97]

Solution: $L = 2$ H, $V = 150$ V, $f = 50$ Hz

The maximum value of the current may be given as:

$$I_0 = \sqrt{2} \times I_{rms} = \sqrt{2} \times \frac{V_{rms}}{X_L} = \sqrt{2} \times \frac{V_{rms}}{2\pi f L} = \sqrt{2} \times \frac{150}{2\pi \times 50 \times 2} = 0.338 \text{ A}$$

Problem 7.33: *Determine the required frequency at which the inductive reactance of* 0.7 *H inductor is* 220 Ω*.*

Solution: $L = 0.7$ H, $X_L = 220\ \Omega$

The expression for the inductive reactance may be given as:

$$X_L = 2\pi f L$$

or, $f = \dfrac{X_L}{2\pi L} = \dfrac{220}{2\pi \times 0.7}$ = 50.02 Hz

Problem 7.34: *Determine the capacitive reactance of a* 5 μ*F capacitor, when it is connected across a supply source of frequency: i*) 50 *Hz, ii*) 10^6 *Hz.* [Haryana 1994-95]

Solution: C = 5 μF, f_1 = 50 Hz, $f_2 = 10^6$ Hz

The capacitive reactance may be given as:

$$X_{C1} = \frac{1}{2\pi f_1 C} = \frac{1}{2\pi \times 50 \times 5 \times 10^{-6}} = 636.62\ \Omega$$

and, $$X_{C2} = \frac{1}{2\pi f_2 C} = \frac{1}{2\pi \times 10^6 \times 5 \times 10^{-6}} = 31.83\ \text{m}\Omega$$

Problem 7.35: *A capacitor has a capacitance of* $\dfrac{1}{\pi}$ μ*F. Determine its reactance, when it is connected across a supply source of frequency: i*) 50 *Hz, ii*) 10^6 *Hz.*

Solution: $C = \dfrac{1}{\pi}$ μF, f_1 = 50 Hz, $f_2 = 10^6$ Hz

The capacitive reactance may be given as:

$$X_{C1} = \frac{1}{2\pi f_1 C} = \frac{1}{2\pi \times 50 \times (1/\pi) \times 10^{-6}} = 10\ \text{k}\Omega$$

and, $$X_{C2} = \frac{1}{2\pi f_2 C} = \frac{1}{2\pi \times 10^6 \times (1/\pi) \times 10^{-6}} = 0.5\ \Omega$$

Problem 7.36: *A* 1.5 μ*F capacitor has a capacitive reactance of* 12 Ω*. Determine the frequency of the supply source. If the frequency of the supply source is doubled, determine the new capacitive reactance.*

Solution: C = 1.5 μF, X_{C1} = 12 Ω, $f_2 = 2 f_1$

The expression for the capacitive reactance offered by a capacitor to the flow of current may be given as:

$$X_C = \frac{1}{2\pi f C}$$

or, $$f = \frac{1}{X_C \times 2\pi C} = \frac{1}{12 \times 2\pi \times 1.5 \times 10^{-6}} = 8841.9\ \text{Hz}$$

If the frequency got doubled, the capacitive reactance (being $X_C \propto \dfrac{1}{f}$) becomes halved of the earlier value.

So, $$X_{C2} = \frac{X_C}{2} = \frac{12}{2} = 6\ \Omega$$

Problem 7.37: *A* 10 μ*F capacitor is connected to an oscillator with an output voltage e* = 10 *sin* ω*t Volts. If the angular frequency* ω = 10 *rad/sec, determine the peak current in the circuit.*

Solution: C = 10 μF, $e = 10 \sin \omega t$ Volts ω = 10 rad/sec

The value of peak current flowing through the circuit may be given as:

$$I_0 = \frac{E_0}{X_C} = \frac{E_0}{(1/\omega C)} = \omega\, C\, E_0 = 10 \times 10 \times 10^{-6} \times 10 = 1 \times 10^{-3}\text{ A} = 1\text{ mA}$$

Problem 7.38: *A capacitor has a capacitive reactance of* 100 Ω *at* 50 *Hz. Determine the reactance at a frequency of* 125 *Hz.*

Solution: $X_{C1} = 100\ \Omega$ (at $f_1 = 50$ Hz), $f_2 = 125$ Hz

We know that the capacitive reactance is inversely proportional to the supply frequency,

i.e. $X_C \propto \dfrac{1}{f}$ So, $\dfrac{X_{C2}}{X_{C1}} = \dfrac{f_1}{f_2}$

or, $X_{C2} = \dfrac{f_2}{f_1} \times X_{C1} = \dfrac{50}{125} \times 100 = 40\ \Omega$

7.11 Resistance – Inductance (*R*-*L*) Series Circuit: Refer to Fig. 7.15 (a), which shows an *R*-*L* series circuit in which a resistor and an inductor are connected in series carrying the same current *I*. So, the current vector is same for both the elements [*note the direction of current vector drawn above resistor and inductor in Fig.* 7.15 (*a*) *is same*]. Voltage across the resistor (V_R) is co-phasor with the current vector [*see above the resistor in the Fig.* 7.15 (*a*)], and voltage across the inductor (V_L) is 90° ahead of current vector [*see above the inductor in the Fig.* 7.15 (*a*)], as current lags the voltage by 90° in a pure inductor. Now, a phasor diagram of three voltages in the circuit *V*, V_R and V_L may be drawn as in Fig. 7.15 (b), known as "***Voltage Triangle***". The vectors *I* and V_R are in phase, V_L is 90° ahead of *I* and the resultant of V_R and V_L is *V*. Angle between the resultant (applied) voltage *V* and *I* is ***power factor angle*** (ϕ); the line current *I* is lagging behind the supply voltage *V* by an angle ϕ. So, if supply voltage is given as:

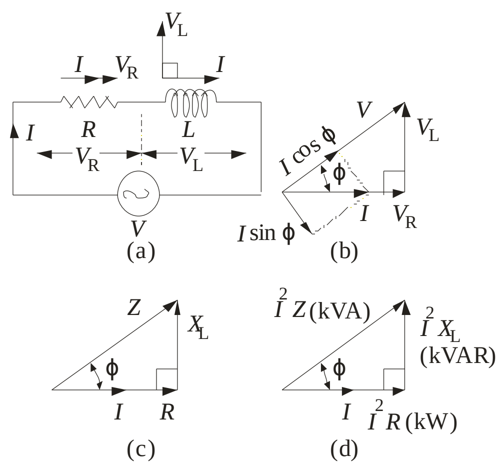


Fig. 7.15

$$v = V_0 \sin \omega t \tag{7.44}$$

Circuit current may be given as: $i = I_0 \sin(\omega t - \phi)$ (7.45)

Power Consumed in a Resistance – Inductance (*R*-*L*) Series Circuit: The instantaneous power in the circuit may be given by the product of instantaneous voltage and instantaneous current:

$$p = v\, i = V_0 \sin \omega t \times I_0 \sin(\omega t - \phi)$$

$$= \frac{V_0 I_0}{2}\,[\cos\{\omega t - (\omega t - \phi)\} - \cos\{\omega t + (\omega t - \phi)\}] = \frac{V_0 I_0}{2}\,[\cos\phi - \cos(2\omega t - \phi)]$$

Now, $$P_{avg} = \frac{1}{\pi} \times \int_0^{\pi} \left[\frac{V_0 I_0}{2}\{\cos\phi - \cos(2\omega t - \phi)\}\right] d(\omega t) = \frac{V_0 I_0}{2\pi} \times \left[(\omega t)\cos\phi - \frac{\sin(2\omega t - \phi)}{2}\right]_0^{\pi}$$

$$= \frac{V_0 I_0}{2\pi} \times \left[\left(\pi \cos\phi - \frac{\sin(2\pi - \phi)}{2}\right) - \left(0 \times \cos\phi - \frac{\sin(2 \times 0 - \phi)}{2}\right)\right]$$

or, $$P_{avg} = \frac{V_0 I_0}{2\pi} \times \left[\left(\pi \cos\phi - \frac{(-\sin\phi)}{2}\right) - \left(0 - \frac{(-\sin\phi)}{2}\right)\right]$$

$$= \frac{V_0 I_0}{2\pi} \times \left[\pi \cos\phi + \frac{\sin\phi}{2} - \frac{\sin\phi}{2}\right] = \frac{V_0 I_0}{2} \times \cos\phi = \frac{V_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}} \times \cos\phi$$

So, $P_{avg} = V\,I \cos\phi$ Watts (7.46)

:- cos ϕ* (*always,* **– 1 < cos ϕ < + 1)** ***is the factor by which the power is being reduced in an A.C. circuit than that in case of a D.C. circuit **(*V I* Watts),** ***that is why*** **cos ϕ** ***is known as "Power Factor*** **(*P.F.*)".**

If the circuit current I, shown in the phasor diagram in Fig. 7.15 (b), is resolved along the supply voltage ($I \cos\phi$) and along a perpendicular direction to supply voltage ($I \sin\phi$). The reader may observe from the equation (7.46) that:

i) *The component I cos ϕ is responsible for the power losses in the circuit, hence is known as* ***Watt-Full Current****.*

ii) On the other hand, *the component I sin ϕ is not responsible for any power loss in the circuit, hence is known as* ***Watt-Less Current****.*

Impedance Triangle: If we divide the voltage triangle [Fig. 7.15 (b)] by the circuit current I, we will get another similar triangle [Fig. 7.15 (c)] known as "***Impedance Triangle***". Refer to *impedance triangle* in Fig. 7.15 (c): (This *impedance triangle* along with *voltage triangle* and *power triangle* is very handy, tricky and useful for solving the numerical problems of single-phase *A.C.* circuits).

Clearly, $R = Z \cos\phi$, and, $X_L = Z \sin\phi$ [Always remember X_L is (+)ve] (7.47)

$$Z = (R + i\,X_L) \quad \text{and,} \quad Z = \sqrt{R^2 + X_L^2} \quad \text{and,} \quad \phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \cos^{-1}\left(\frac{R}{Z}\right) \qquad (7.48)$$

Power Triangle: If we multiply the voltage triangle [Fig. 7.15 (b)] by the circuit current I, we will get another similar triangle [Fig. 7.15 (d)] known as **"*Power Triangle*"**. Refer to power triangle in Fig. 7.15 (d): (*This power triangle is very important. Also it is very handy, tricky and useful for solving the numerical problems of single-phase A.C. circuits.*)

Apparent Power ($I^2 Z$ or *VI*, its unit is kVA): The hypotenuse of the power triangle is the apparent power used in the circuit. Its unit is Volt-Amp (VA) or Kilo-Volt-Amp (kVA).

Apparent Power = $V\,I = I^2 Z$ (kVA) (7.49)

True Power ($I^2 R$ or $V\,I \cos\phi$, its unit is kW): The base of the power triangle is the power dissipated in the resistance and is true / real power, as power dissipates in resistance only. Its unit is Watt (W) or Kilo-Watt (kW).

True Power = $V\,I \cos\phi = I^2 R$ (kW) (7.50)

Reactive Power ($I^2 X_L$ or $V\,I \sin\phi$, its unit is kVAR): The perpendicular of the power triangle is the power stored and returned in the reactance and is known as reactive power as power is not being dissipated in the reactance. Its unit is Volt-Amp-Reactive (VAR) or Kilo-Volt-Amp-Reactive (kVAR).

Reactive Power = $V\,I \sin\phi = I^2 X_L$ (kVAR) (7.51)

It can clearly be observed from above discussion and the power triangle, that:

$$(\text{Apparent Power})^2 = (\text{True Power})^2 + (\text{Reactive Power})^2$$

or, $$(\text{kVA})^2 = (\text{kW})^2 + (\text{kVAR})^2 \qquad (7.52)$$

Problem 7.39: *If an inductor and a resistor are connected in series across a* 12 *V,* 50 *Hz supply source, a current of* 0.5 *A is flowing through them. The phase difference between the applied voltage and the circuit current is* $\frac{\pi}{3}$ *rad. Determine the value of R and L in the circuit.* [CBSE 2005-06]

Solution: **An *R-L* Series Circuit:** $V = 12$ V, $f = 50$ Hz, $I = 0.5$ A, $\phi = \frac{\pi}{3}$ rad = 60°

The impedance of the circuit may be given as:

$$Z = \frac{V}{I} = \frac{12}{0.5} = 24\ \Omega$$

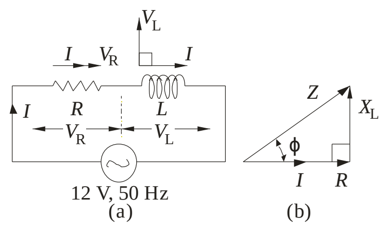


Fig. 7.16

So, the circuit elements (resistance and inductor in the circuit) may respectively be given, using the impedance triangle, as:

$$R = Z\cos\phi = 24 \times \cos 60° = 24 \times 0.5 = 12\ \Omega$$

and, $$X_L = Z\sin\phi = 24 \times \sin 60° = 24 \times \frac{\sqrt{3}}{2} = 20.785\ \Omega = 2\,\pi\, f\, L$$

So, $$L = \frac{X_L}{2\pi f} = \frac{20.785}{2\pi \times 50} = 66.16 \text{ mH}$$

Problem 7.40: *A lamp of resistance* 10 Ω*, in series with an inductor L, is connected across a supply source of* 100 *V,* 50 *Hz. If the phase angle between the applied voltage and the circuit current is* $\frac{\pi}{4}$ *rad, determine the value of inductance* (*L*)*. Also determine the value of current flowing through the circuit.* [CBSE 2000-01]

Solution: **An *R-L* Series Circuit:** $R = 10\ \Omega$, $V = 10$ V, $V = 100$ V, $f = 50$ Hz, $\phi = \frac{\pi}{4}$ rad

Using the impedance triangle:

$$X_L = 2\,\pi\, f\, L = R\tan\phi = 10 \times \tan 45° = 10\ \Omega$$

or, $$L = \frac{10}{2\pi \times 50} = 31.83 \text{ mH}$$

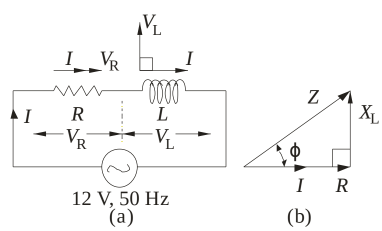


Fig. 7.16

The current flowing through the circuit may be given as:

$$I = \frac{V}{Z} = \frac{100}{\sqrt{(10)^2 + (10)^2}} = \frac{100}{10\sqrt{2}} = 5\sqrt{2}\ \text{A} = 7.071\ \text{A}$$

Problem 7.41: *A coil having a resistance of* 300 Ω *and inductance* 1 *H is connected across an alternating supply source of* $900\sqrt{2}$ *V,* $\frac{300}{2\pi}$ *Hz. Determine the current flowing through the circuit and the phase difference between the supply voltage and the current.*

Solution: **An *R*-*L* Series Circuit:** $R = 300\ \Omega$, $L = 1$ H, $V = 900\sqrt{2}$ V, $f = \dfrac{300}{2\pi}$ Hz

The inductive reactance of the coil may be given as:

$$X_L = 2\pi f L = 2\pi \times \frac{300}{2\pi} \times 1 = 300\ \Omega$$

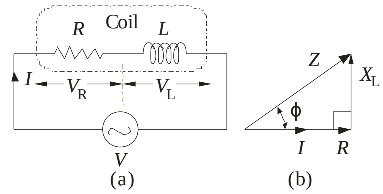


Fig. 7.17

The current flowing through the circuit may be given as:

$$I = \frac{V}{Z} = \frac{900\sqrt{2}}{\sqrt{(300)^2 + (300)^2}} = \frac{900\sqrt{2}}{300\sqrt{2}} = 3\ \text{A}$$

The phase difference between the supply voltage and the current may be given as:

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{300}{300}\right) = 45°$$

Problem 7.42: *A coil when connected across a* 10 *V d.c. supply draws a current of* 2 *A. When the same coil is connected across* 10 *V,* 50 *Hz a.c. supply source, it draws a current of* 1 *A. Explain the reason why it draws a smaller current on a.c. and determine the parameter due to which it is drawing a smaller current from a.c.* [CBSE 2002-03]

Solution: **An *R*-*L* Series Circuit:** $V_{d.c.} = 10$ V (d.c.), $I_{d.c.} = 2$ A, $V_{a.c.} = 10$ V (50 Hz, a.c.), $I_{a.c.} = 1$ A

A coil connected across a d.c. source acts as a pure resistor at steady state, as the inductance of the coil behaves as short circuit for d.c. supply source at steady state of the circuit, while the same coil connected across an a.c. supply source acts as impedance due to the resistance as well as the inductive reactance offered by the coil to flow of the current. So, a coil draws a smaller current, when it is connected across an a.c. supply source.

The resistance of the coil may be given as:

$$R = \frac{V_{d.c.}}{I_{d.c.}} = \frac{10}{2} = 5\ \Omega$$

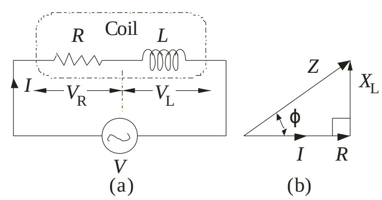


Fig. 7.18

The impedance of the coil may be given as:

$$Z = \frac{V_{a.c.}}{I_{a.c.}} = \frac{10}{1} = 10\ \Omega$$

So, the inductive reactance offered by the coil to the flow of current may be given as:

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{(10)^2 - (5)^2} = \sqrt{75} = 8.66\ \Omega = 2\pi f L$$

or, $$L = \frac{8.66}{2\pi f} = \frac{8.66}{2\pi \times 50} = 27.57\ \text{mH}$$

Problem 7.43: *An* 80 *V,* 800 *W heater is to be operated on a* 100 *V,* 50 *Hz supply. Determine the inductance of the choke coil required for the purpose.* [CBSE 1990-91, Haryana 2001-02]

Solution: Heater = 80 V, 800 W, $V = 100$ V, $f = 50$ Hz

The choke coil is required in series with the heater in order to share the excess voltage above the rated voltage of the heater. The setup is shown in the Fig. 7.19 (a), and the corresponding phasor diagram is shown in the Fig. 7.19 (b).

The rated current of the heater may be given as:

$$I_{rated} = \frac{P_{rated}}{V_{rated}} = \frac{800}{80} = 10 \text{ A}$$

The voltage across the choke coil, according to phasor diagram shown in the Fig. 7.19 (b), may be given as:

$$V_L = \sqrt{V^2 - V_R^2} = \sqrt{(100)^2 - (80)^2} = 60 \text{ V}$$

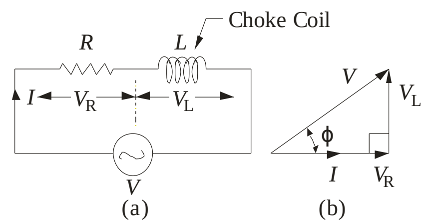


Fig. 7.19

The inductive reactance of the choke coil required for the purpose may be given as:

$$X_L = \frac{V_L}{I_{rated}} = \frac{60}{10} = 6\ \Omega = 2\,\pi\,f\,L$$

So, $$L = \frac{6}{2\pi f} = \frac{6}{2\pi \times 50} = 19.1 \text{ mH}$$

Problem 7.44: *A student connects a long air core coil of manganin wire to a* 100 *V d.c. source and records a current of* 1.5 *A. When the same coil is connected across* 100 *V,* 50 *Hz a.c. source the current reduces to* 1 *A. i*) *Explain the reason for this observation. ii*) *Determine the value of the reactance and inductance of the coil.* [CBSE 1993-94]

Solution: **An *R*-*L* Series Circuit:** $V_{d.c.}$ = 100 V (d.c.), $I_{d.c.}$ = 1.5 A, $V_{a.c.}$ = 100 V (50 Hz, a.c.), $I_{a.c.}$ = 1 A

A coil connected across a d.c. source acts as a pure resistor at steady state, as the inductance of the coil behaves as short circuit for d.c. supply source at steady state of the circuit, while the same coil connected across an a.c. supply source acts as impedance due to the resistance as well as the inductive reactance offered by the coil to flow of the current. So, a coil draws a smaller current, when it is connected across an a.c. supply source.

The resistance of the coil may be given as:

$$R = \frac{V_{d.c.}}{I_{d.c.}} = \frac{100}{1.5} = 66.667\ \Omega$$

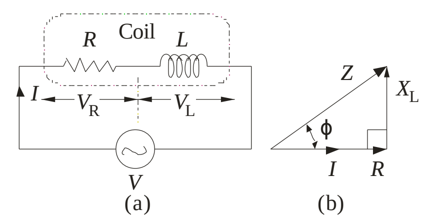


Fig. 7.20

The impedance of the coil may be given as:

$$Z = \frac{V_{a.c.}}{I_{a.c.}} = \frac{100}{1} = 100\ \Omega$$

So, the inductive reactance offered by the coil to the flow of current may be given as:

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{(100)^2 - (66.667)^2} = 74.534\ \Omega = 2\,\pi\,f\,L$$

or, $$L = \frac{75.535}{2\pi f} = \frac{75.535}{2\pi \times 50} = 240.435 \text{ mH}$$

Problem 7.45: *When a* 200 *V d.c. supply is connected across a coil, a current of* 2 *A flows through it. When the same coil is connected across a* 200 *V,* 50 *Hz a.c. supply the current through the coil is observed to be* 1 *A only. Determine the resistance, impedance and inductance of the coil.* [CBSE 1994-95]

Solution: **An *R*-*L* Series Circuit:** $V_{d.c.}$ = 200 V (d.c.), $I_{d.c.}$ = 2 A, $V_{a.c.}$ = 200 V (50 Hz, a.c.), $I_{a.c.}$ = 1 A

A coil connected across a d.c. source acts as a pure resistor at steady state, as the inductance of the coil behaves as short circuit for d.c. supply source at steady state of the circuit, while the same coil connected across an a.c. supply source acts as impedance due to the resistance as well as the inductive reactance offered by the coil to flow of the current. So, a coil draws a smaller current, when it is connected across an a.c. supply source.

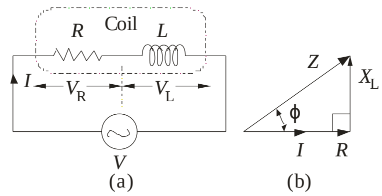


Fig. 7.21

The resistance of the coil may be given as:

$$R = \frac{V_{d.c.}}{I_{d.c.}} = \frac{200}{2} = 100\ \Omega$$

The impedance of the coil may be given as:

$$Z = \frac{V_{a.c.}}{I_{a.c.}} = \frac{200}{1} = 200\ \Omega$$

So, the inductive reactance offered by the coil to flow of current may be given as:

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{(200)^2 - (100)^2} = 173.21\ \Omega = 2\,\pi\,f\,L$$

or, $$L = \frac{173.21}{2\pi\,f} = \frac{173.21}{2\,\pi \times 50} = 551.345\ \text{mH}$$

Problem 7.46: *A* 60 *V,* 10 *W lamp is to be operated on* 100 *V,* 60 *Hz mains. i*) *Determine the inductance of the choke coil required for the purpose. ii*) *If a resistance is to be used in place of the choke coil to achieve the same result, determine its value.* [CBSE 1996-97]

Solution: Lamp = 60 V, 10 W, V = 100 V, f = 60 Hz

The rated current of the lamp may be given as:

$$I_{rated} = \frac{P_{rated}}{V_{rated}} = \frac{10}{60} = 0.167\ \text{A}$$

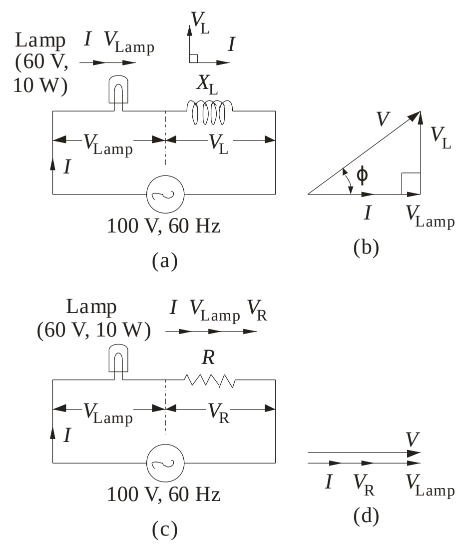


Fig. 7.22

Using Choke Coil: The connection diagram is shown in the Fig. 7.22 (a), and the voltage across the choke coil, according to phasor diagram shown in the Fig. 7.22 (b), may be given as:

$$V_L = \sqrt{V^2 - V_{Lamp}^2} = \sqrt{(100)^2 - (60)^2} = 80\ \text{V}$$

The inductive reactance of the coil may be given as:

$$X_L = \frac{V_L}{I_{rated}} = \frac{80}{0.167} = 479.04\ \Omega = 2\,\pi\,f\,L$$

So, $$L = \frac{479.04}{2\pi\,f} = \frac{479.04}{2\,\pi \times 60} = 1.272\ \text{H}$$

Using Resistor: The connection diagram is shown in the Fig. 7.22 (c), and the voltage across the resistor R, according to phasor diagram shown in the Fig. 7.22 (d), may be given as:

$$V_R = V - V_{Lamp} = 100 - 60 = 40\ \text{V}$$

The required resistance may be given as:

$$R = \frac{V_R}{I_{rated}} = \frac{40}{0.167} = 239.52\ \Omega$$

Problem 7.47: *A* 12 Ω *resistor in series with an inductance of* $\frac{0.05}{\pi}$ *H is connected across a* 130 *V,* 50 *Hz a.c. source. Determine: i*) *the current flowing through the circuit, ii*) *phase difference between the applied voltage and the current flowing through the circuit.* [Haryana 2002-03]

Solution: $R = 12\ \Omega$, $L = \frac{0.05}{\pi}$ H, $V = 130$ V, $f = 50$ Hz

The inductive reactance offered by the inductance to the flow of current may be given as:

$$X_L = 2\,\pi\, f\, L = 2\,\pi \times 50 \times \frac{0.05}{\pi} = 5\ \Omega$$

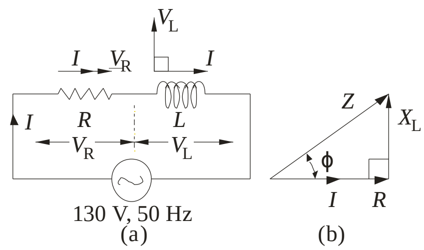


Fig. 7.23

So, the current flowing through the circuit may be given as:

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + X_L^2}} = \frac{130}{\sqrt{(12)^2 + (5)^2}} = 10\ \text{A}$$

The phase difference between the applied voltage and the current flowing through the circuit may be given as:

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{5}{12}\right) = 22.62°$$

Problem 7.48: *The circuit shown in the Fig.* 7.24 *has* $V_R = 160$ *V and* $V_L = 120$ *V. Determine the value of applied voltage* (*V*) *across the circuit. If the current flowing in the circuit is* 1 *A, determine the impedance of the circuit. If a d.c. supply source is applied across the circuit to flow the same current through the circuit, what must be the potential difference across the circuit.*

Solution: $V_R = 160$ V, $V_L = 120$ V, $I = 1$ A

The applied voltage (*V*) across the circuit, according to the phasor diagram shown in the Fig. 7.24 (b), may be given as:

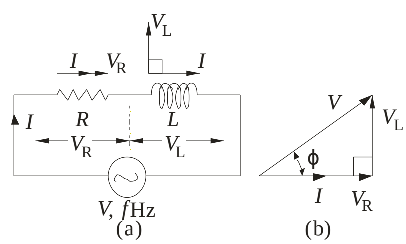


Fig. 7.24

$$V = \sqrt{V_R^2 + V_L^2} = \sqrt{(160)^2 + (120)^2} = 200\ \text{V}$$

The impedance of the circuit may be given as:

$$Z = \frac{V}{I} = \frac{200}{1} = 200\ \Omega$$

The potential difference across the circuit, in case the same d.c. current is flowing through the circuit, may be given as:

$$V = V_R = 160\ \text{V}$$

(Since, there will be no induced emf across the inductor because of $f = 0$ Hz and $X_L = 0\ \Omega$ due to d.c. current)

Problem 7.49: *The circuit shown in the Fig.* 7.25 *has* $V_R = 90$ *V and* $V_L = 120$ *V. Determine the value of applied voltage* (*V*) *across the circuit. If the current flowing in the circuit is* 3 *A determine the impedance of the circuit and the phase angle between the applied voltage and the current flowing through the circuit.* [CBSE 2003-04]

Solution: $V_R = 90$ V, $V_L = 120$ V, $I = 3$ A

The applied voltage (V) across the circuit, according to the phasor diagram shown in the Fig. 7.25 (b), may be given as:

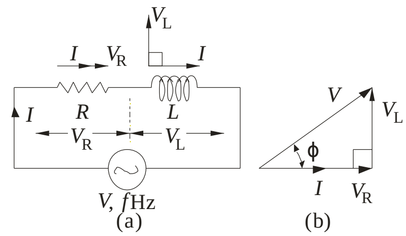


Fig. 7.25

$$V = \sqrt{V_R^2 + V_L^2} = \sqrt{(90)^2 + (120)^2} = 150 \text{ V}$$

The impedance of the circuit may be given as:

$$Z = \frac{V}{I} = \frac{150}{3} = 50\ \Omega$$

The phase angle between the applied voltage and the current flowing through the circuit may be given as:

$$\phi = \tan^{-1}\left(\frac{V_L}{V_R}\right) = \tan^{-1}\left(\frac{120}{90}\right) = 53.13°$$

Problem 7.50: *Determine the impedance of a coil of resistance* 3 Ω *and reactance* 4 Ω.

Solution: $R = 3\ \Omega$, $X_L = 4\ \Omega$

The impedance of the coil may be given as:

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{(3)^2 + (4)^2} = 5\ \Omega$$

Problem 7.51: *An inductive coil has a resistance of* 100 Ω. *When an a.c. signal of frequency* 1000 *Hz is applied to the coil, the applied voltage leads the resultant current by an angle of* 45°. *Determine the self inductance of the coil.* [CBSE 1997-98]

Solution: $R = 100\ \Omega$, $f = 1000$ Hz, $\phi = 45°$ (current is lagging)

The reactance of the coil, according to the impedance triangle, may be given as:

$$X_L = R \tan\phi = 100 \tan 45° = 100\ \Omega = 2\pi f L$$

or, $$L = \frac{100}{2\pi f} = \frac{100}{2\pi \times 1000} = 15.92 \text{ mH}$$

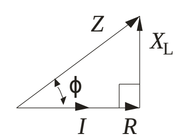


Fig. 7.26

Problem 7.52: *An a.c. source of* 100 *V,* 50 *Hz is connected across* 20 Ω *resistor and* 20 *mH inductor connected in series. Determine: i) impedance of the circuit, ii) r.m.s. current in the circuit.* [CBSE 1992-93]

Solution: $V = 100$ V (r.m.s.), $f = 50$ Hz, $R = 20\ \Omega$, $L = 20$ mH

The impedance of the circuit may be given as:

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (2\pi f L)^2}$$

$$= \sqrt{(20)^2 + (2\pi \times 50 \times 20 \times 10^{-3})^2} = 20.964\ \Omega$$

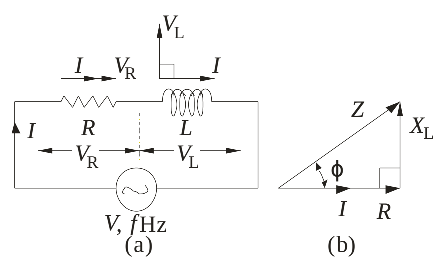


Fig. 7.27

The r.m.s value of the current through the circuit may be given as:

$$I = \frac{V}{Z} = \frac{100}{20.964} = 4.77 \text{ A}$$

Problem 7.53: *An a.c. source of* 100 *V,* 50 *Hz is connected to a series combination of an inductor of* 100 *mH and a resistance of* 20 Ω*. Determine the magnitude and phase angle of the current flowing through the circuit.* [CBSE 1990-91]

Solution: $V = 100$ V (r.m.s.), $f = 50$ Hz, $L = 100$ mH, $R = 20\ \Omega$

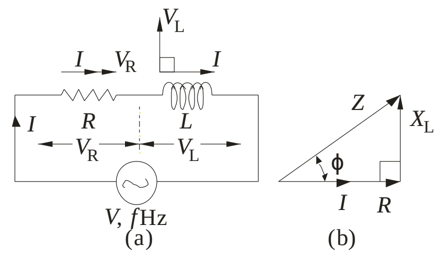


Fig. 7.27

The reactance offered by the inductor may be given as:

$$X_L = 2\,\pi\,f\,L = 2\,\pi \times 50 \times 0.100 = 31.42\ \Omega$$

The impedance of the circuit may be given as:

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{(20)^2 + (31.42)^2}$$

$$= 37.245\ \Omega$$

The magnitude of the current flowing through the circuit may be given as:

$$I = \frac{V}{Z} = \frac{100}{37.245} = 2.685\ \text{A}$$

The phase angle of the current flowing through the circuit may be given as:

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{31.42}{20}\right) = 57.52° \text{ (lagging)}$$

Problem 7.54: *A current of* 11 *A flows through a coil, when connected across a* 110 *V d.c. source. When the same coil is connected across a* 110 *V,* 50 *Hz a.c. source, the current through the circuit is found to be* 0.5 *A only. Determine: i*) *impedance, ii*) *resistance, iii*) *inductance of the coil.* [CBSE 1991-92]

Solution: $I_{d.c.} = 11$ A, $V_{d.c.} = 110$ V, $V_{a.c.} = 110$ V, $f = 50$ Hz, $I = 0.5$ A

A coil connected across a d.c. source acts as a pure resistor at steady state, as the inductance of the coil behaves as short circuit for d.c. supply source at steady state of the circuit, while the same coil connected across an a.c. supply source acts as impedance due to the resistance as well as the inductive reactance offered by the coil to flow of the current. So, a coil draws a smaller current, when it is connected across an a.c. supply source.

The resistance of the coil may be given as:

$$R = \frac{V_{d.c.}}{I_{d.c.}} = \frac{110}{11} = 10\ \Omega$$

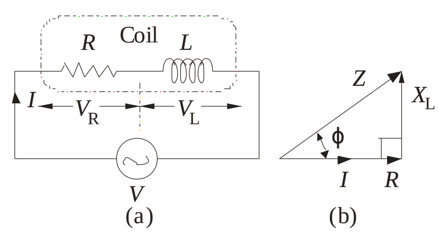


Fig. 7.29

The impedance of the coil may be given as:

$$Z = \frac{V_{a.c.}}{I_{a.c.}} = \frac{110}{0.5} = 220\ \Omega$$

The reactance of the coil may be given as:

$$X_L = \sqrt{(220)^2 - (10)^2} = 219.77 = 2\,\pi\,f\,L$$

or, $$L = \frac{219.77}{2\,\pi\,f} = \frac{219.77}{2\,\pi \times 50} = 0.7\ \text{H} = 700\ \text{mH}$$

Problem 7.55: *An arc lamp takes a rated current of* 10 *A at a rated voltage of* 80 *V. Determine the inductance, which must be connected in series with the arc lamp to work the lamp on the correct voltage while connected to a* 220 *V,* 50 *Hz a.c. source.*

Solution: I_{rated} = 10 A, V_{rated} = 80 V, V = 220 V, f = 50 Hz

The voltage across the inductor connected in series with the arc lamp, according to the phasor diagram in Fig. 7.30 (b), may be given as:

$$V_L = \sqrt{V^2 - V_{Arc}^2} = \sqrt{(220)^2 - (80)^2}$$

$$= 204.94 \text{ V}$$

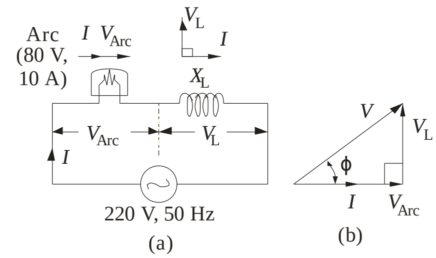


Fig. 7.30

So, the required inductive reactance in series with the arc lamp may be given as:

$$X_L = \frac{V_L}{I} = \frac{204.94}{10} = 20.494\ \Omega = 2\,\pi\, f\, L$$

or, $$L = \frac{20.494}{2\pi f} = \frac{20.494}{2\pi \times 50} = 0.06523 \text{ H} = 65.23 \text{ mH}$$

Problem 7.56: *A current of* 2 *A is flowing through an R-L series circuit shown in the Fig.* 7.31. *If the voltage across the resistor* (*R*) *is* 100 *V and that across the inductor* (*L*) *is* 240 *V, determine the voltage of the a.c. source and the impedance of the circuit.*

Solution: I = 2 A, V_R = 100 V, V_L = 240 V

The voltage across the supply source may, according to the phasor diagram drawn in the Fig. 7.31 (b), may be given as:

$$V = \sqrt{V_R^2 + V_L^2} = \sqrt{(100)^2 + (240)^2} = 260 \text{ V}$$

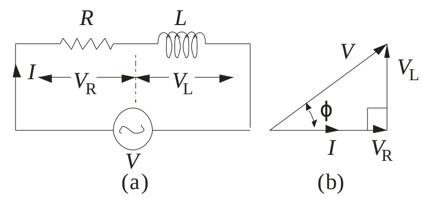


Fig. 7.31

The impedance of the circuit may be given as:

$$Z = \frac{V}{I} = \frac{260}{2} = 130\ \Omega$$

Problem 7.57: *Determine the impedance and the r.m.s. value of the current flowing through an R-L series circuit in which a resistance of* 30 Ω *and an inductor of* 100 *mH are connected in series across a voltage source of v* = 200 *sin* 400*t Volts.*

Solution: R = 30 Ω, L = 100 mH, v = 200 sin 400t Volts

Comparing the given expression of emf with the standard equation: $v = V_0 \sin \omega t$

V_0 = 200 V, ω = 400 rad/sec

The impedance of the circuit may be given as:

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (\omega L)^2}$$

$$= \sqrt{(30)^2 + (400 \times 0.100)^2} = 50\ \Omega$$

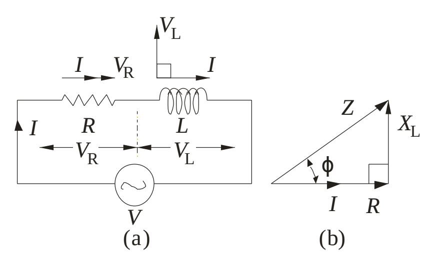


Fig. 7.32

The r.m.s. value of the current flowing through the circuit may be given as:

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{V_0}{\sqrt{2} \times Z} = \frac{200}{\sqrt{2} \times 50} = 2\sqrt{2}\ \text{ A} = 2.828 \text{ A}$$

Problem 7.58: *The current flowing through an R-L series circuit, connected across an a.c. source of* 200 *V,* 50 *Hz, is* 1 *A and the voltage across the resistor is* 120 *V. Determine: i*) *the voltage across the inductor* (*L*), *ii*) *the impedance of the circuit, iii*) *the value of the resistance, iv*) *the reactance and inductance of the coil.*

Solution: $V = 200$ V, $f = 50$ Hz, $I = 1$ A, $V_R = 120$ V

The voltage across the inductor may, according to the phasor diagram drawn in the Fig. 7.33 (b), may be given as:

$$V_L = \sqrt{V^2 - V_R^2} = \sqrt{(200)^2 - (120)^2} = 160 \text{ V}$$

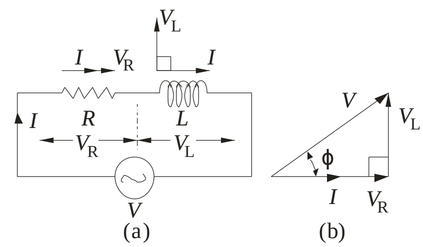


Fig. 7.33

The impedance of the circuit may be given as:

$$Z = \frac{V}{I} = \frac{200}{1} = 200\ \Omega$$

The value of the resistance may be given as:

$$R = \frac{V_R}{I} = \frac{120}{1} = 120\ \Omega$$

The value of the reactance of the coil may be given as:

$$X_L = \frac{V_L}{I} = \frac{160}{1} = 160\ \Omega$$

The value of the inductance of the coil may be given as:

$$L = \frac{X_L}{2\pi f} = \frac{160}{2\pi \times 50} = 509.3 \text{ mH}$$

Problem 7.59: *A circuit containing a resistor of* 50 Ω *and an inductor of* $\frac{1}{\pi}$ *H in series is connected across a* 200 *V,* 60 *Hz a.c. mains. Determine: i*) *the reactance and the impedance of the circuit, ii*) *the magnitude and phase angle of the current flowing through the circuit.*

Solution: $R = 50\ \Omega$, $L = \frac{1}{\pi}$ H, $V = 200$ V, $f = 60$ Hz

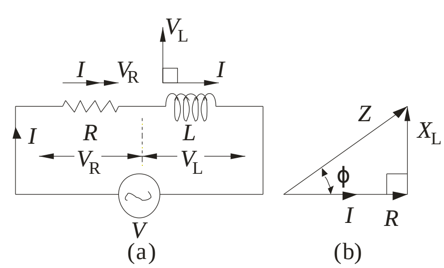


Fig. 7.34

The inductive reactance of the circuit may be given as:

$$X_L = 2\,\pi\, f\, L = 2\,\pi \times 60 \times \frac{1}{\pi} = 120\ \Omega$$

The impedance of the circuit may be given as:

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{(50)^2 + (120)^2} = 130\ \Omega$$

The magnitude of the current flowing through the circuit may be given as:

$$I = \frac{V}{Z} = \frac{200}{130} = 1.538 \text{ A}$$

The phase angle of the current flowing through the circuit may be given as:

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{120}{50}\right) = 67.38° \text{ (lagging)}$$

Problem 7.60: *An a.c. circuit consists of a* 220 *V,* 50 *Hz supply source connected across a* 100 Ω *resistor. Determine the value of inductance to be connected in the circuit in series with the resistance, so that the current is reduced to half of the original value.*

Solution: $V = 220$ V, $f = 50$ Hz, $R = 100\ \Omega$, $I_2 = \dfrac{I_1}{2}$

The current flowing through the resistor connected alone across the source may be given as:

$$I_1 = \frac{V}{R} = \frac{220}{100} = 2.2 \text{ A}$$

Let the inductor connected in series be L. The current flowing through the series combination may now be given as:

$$I_2 = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (2\pi f L)^2}}$$

$$= \frac{220}{\sqrt{(100)^2 + (2\pi \times 50 \times L)^2}} = \frac{2.2}{2}$$

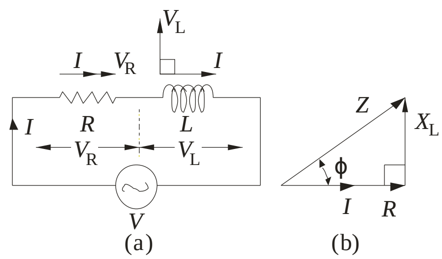


Fig. 7.35

or, $$(100)^2 + (2\,\pi \times 50 \times L)^2 = \left(\frac{220}{1.1}\right)^2 = (200)^2$$

or, $$L = \frac{\sqrt{(200)^2 - (100)^2}}{2\pi \times 50} = 551.33 \text{ mH}$$

Problem 7.61: *A long solenoid draws a steady state current of* 2 *A when connected across a* 12 *V d.c. source. If the same solenoid is connected across a* 12 *V,* 50 *Hz a.c. source, it draws a current of* 1 *A from the source. Determine the inductance of the solenoid.*

Solution: $I_{d.c.} = 2$ A, $V_{d.c.} = 12$ V, $V_{a.c.} = 12$ V, $f = 50$ Hz, $I_{a.c.} = 1$ A

A solenoid connected across a d.c. source acts as a pure resistor at steady state, as the inductance of the solenoid behaves as short circuit for d.c. supply source at steady state of the circuit, while the same solenoid connected across an a.c. supply source acts as impedance due to the resistance as well as the inductive reactance offered by the solenoid to flow of the current. So, a solenoid draws a smaller current, when it is connected across an a.c. supply source.

The resistance of the solenoid may be given as:

$$R = \frac{V_{d.c.}}{I_{d.c.}} = \frac{12}{2} = 6\ \Omega$$

The impedance of the solenoid may be given as:

$$Z = \frac{V_{a.c.}}{I_{a.c.}} = \frac{12}{1} = 12\ \Omega$$

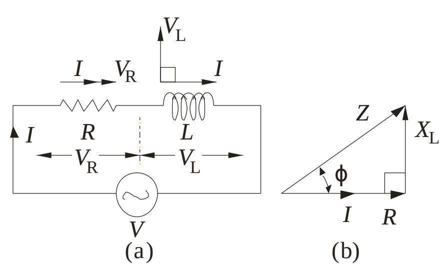


Fig. 7.36

So, the inductive reactance of the coil, according to the phasor diagram drawn in the Fig. 7.36 (b), may be given as:

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{(12)^2 - (6)^2} = 10.392\ \Omega = 2\,\pi f L$$

or, $$L = \frac{10.392}{2\pi f} = \frac{10.392}{2\pi \times 50} = 33.08 \text{ mH}$$

Problem 7.62: *A choke coil and a resistor are connected in series across a supply source of* 130 *V,* 50 *Hz. If the potential difference across the resistor is* 50 *V, determine the potential difference across the choke coil.*

Solution: V = 130 V, f = 50 Hz, V_R = 50 V

The potential difference across the choke coil, according to the phasor diagram drawn in the Fig. 7.37 (b), may be given as:

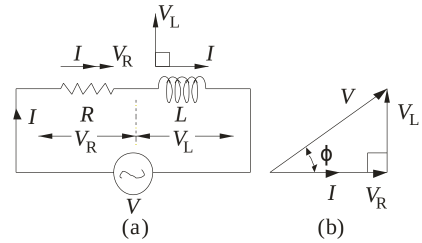


Fig. 7.37

$$V_L = \sqrt{V^2 - V_R^2} = \sqrt{(130)^2 - (50)^2}$$

$$= 120 \text{ V}$$

Problem 7.63: *A source of emf e = 200 sin 377t Volts is applied across a coil of inductance L having a resistance of* 1 Ω*. The maximum value of the current is found to be* 10 *A. Determine the value of self inductance of the coil* (L)*.*

Solution: e = 200 sin 377t Volts, R = 1 Ω, I_0 = 10 A

Comparing the given emf expression with the standard emf equation, we get:

E_0 = 200 V, ω = 377 rad/sec

The expression for the maximum value of the current flowing through the inductive coil may be given as:

$$I_0 = \frac{E_0}{Z} = \frac{E_0}{\sqrt{(R)^2 + (\omega L)^2}}$$

$$= \frac{200}{\sqrt{(1)^2 + (377 \times L)^2}} = 10$$

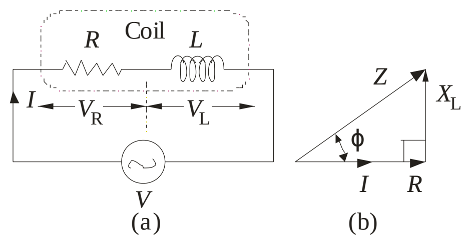


Fig. 7.38

or, $$(1)^2 + (377 \times L)^2 = \left(\frac{200}{10}\right)^2 = (20)^2$$

or, $$L = \frac{\sqrt{(20)^2 - (1)^2}}{377} = 52.984 \text{ mH}$$

Problem 7.64: *An electric circuit containing a resistor R and an inductor L in series has an impedance of* 50 Ω *at* 100 *Hz and an impedance of* 100 Ω *at* 500 *Hz. Determine the value of R and L.*

Solution: Z_1 = 50 Ω (at f_1 = 100 Hz), Z_2 = 100 Ω (at f_2 = 500 Hz)

The expressions for the impedance of the circuit at two different frequencies, according to impedance triangle shown in the Fig. 7.39, may respectively be given as:

$$Z_1 = \sqrt{(R)^2 + (2\pi f L)^2} = 50$$

or, $$(R)^2 + (2\pi \times 100 \times L)^2 = (50)^2$$

or, $$R^2 + 394784.176\, L^2 = 2500 \qquad (7.53)$$

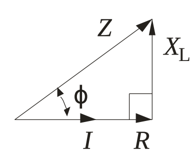


Fig. 7.39

and, $$Z_2 = \sqrt{(R)^2 + (2\pi f L)^2} = 100$$

or, $$(R)^2 + (2\pi \times 500 \times L)^2 = (100)^2$$

or, $$R^2 + 9869604.401\, L^2 = 10000 \qquad (7.54)$$

Equation (7.54) – (7.53):

$$9474820.225\, L^2 = 7500$$

or, $L = \sqrt{\dfrac{7500}{9474820.225}}$ = 28.135 mH

Putting the value of L in equation (7.53):

$$R = \sqrt{2500 - 394784.176 \times (0.028135)^2} = 46.771\ \Omega$$

Problem 7.65: *An R-L circuit, having a resistance of* 30 Ω *and an inductive reactance of* 40 Ω *in series, is connected across a peak emf of* 220 *V. Determine the: i*) *impedance of the circuit, ii*) *phase difference between the applied emf and the resulting current in the circuit, iii*) *the peak value of the current flowing through the circuit.*

Solution: $R = 30\ \Omega$, $X_L = 40\ \Omega$, $E_0 = 220$ V

The impedance of the circuit, according to the impedance triangle shown in the Fig. 7.40 (b), may be given as:

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{(30)^2 + (40)^2} = 50\ \Omega$$

The phase difference between the applied emf and the resulting current in the circuit, according to the impedance triangle shown in the Fig. 7.40 (b), may be given as:

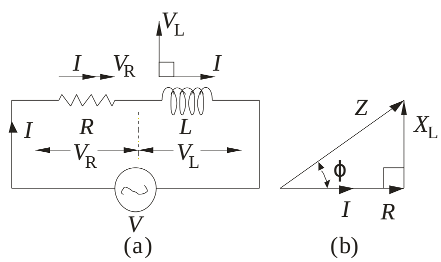


Fig. 7.40

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{40}{30}\right)$$

$$= 53.13° \text{ (lagging)}$$

The peak value of the current flowing through the circuit may be given as:

$$I_0 = \frac{E_0}{Z} = \frac{220}{50} = 4.4 \text{ A}$$

7.12 Resistance – Capacitance (*R*-*C*) Series Circuit: Refer to *Fig.* 7.41 (a), which shows an *R*-*C* series circuit in which a resistor and a capacitor are connected in series carrying the same current *I*. So, the current vector is same for both the elements [*note the direction of current vector drawn above resistor and capacitor in Fig.* 7.41 (*a*) *is same*]. Voltage across the resistor (V_R) is co-phasor with the current vector [*see above the resistor in the Fig.* 7.41 (*a*)], and voltage across the capacitor (V_C) is 90° behind the current vector [*see above the capacitor in the Fig.* 7.41 (*a*)], as current leads the voltage by 90° in a pure capacitor. Now, a phasor diagram of three voltages in the circuit V, V_R and V_C can be drawn as in Fig. 7.41 (b), known as "***Voltage Triangle***". The vectors I and V_R are in same phase, V_C is 90° behind the current I and resultant of V_R and V_C is V. Angle between the resultant (applied) voltage V and the current I is power factor angle, here I is leading the supply voltage V by an angle ϕ. So, if supply voltage is given as:

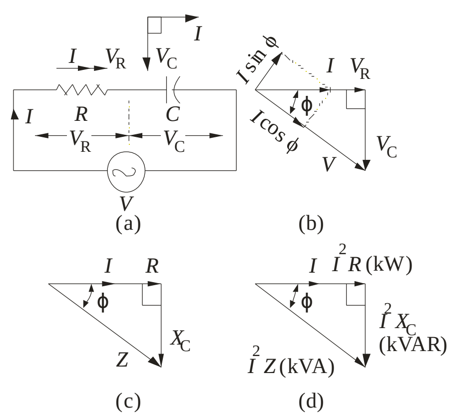


Fig. 7.41

$$v = V_0 \sin \omega t \qquad (7.55)$$

Circuit current may be given by: $i = I_0 \sin(\omega t + \phi)$ (7.56)

Power Consumed in a Resistance-Capacitance (*R*-*C*) Series Circuit: The instantaneous power in the circuit may be given by the product of instantaneous voltage and instantaneous current:

$$p = v \times i = V_0 \sin \omega t \times I_0 \sin (\omega t + \phi)$$

$$= \frac{V_0 I_0}{2} \times [\cos\{\omega t - (\omega t + \phi)\} - \cos\{\omega t + (\omega t + \phi)\}] = \frac{V_0 I_0}{2} \times [\cos \phi - \cos (2\omega t + \phi)]$$

So, $$P_{avg} = \frac{1}{\pi} \times \int_0^{\pi} \frac{V_0 I_0}{2} \times [\cos \phi - \cos (2\omega t + \phi)]\, d(\omega t)$$

$$= \frac{V_0 I_0}{2\pi} \times \left[(\omega t) \times \cos \phi - \frac{\sin (2\omega t + \phi)}{2} \right]_0^{\pi}$$

$$= \frac{V_0 I_0}{2\pi} \times \left[\left\{ (\pi) \times \cos \phi - \frac{\sin (2\pi + \phi)}{2} \right\} - \left\{ (0) \times \cos \phi - \frac{\sin (0 + \phi)}{2} \right\} \right]$$

$$= \frac{V_0 I_0}{2\pi} \times \left[\left\{ (\pi) \times \cos \phi - \frac{\sin \phi}{2} \right\} - \left\{ 0 - \frac{\sin \phi}{2} \right\} \right]$$

$$= \frac{V_0 I_0}{2\pi} \times (\pi) \cos \phi = \frac{V_0 I_0}{2} \times \cos \phi = \frac{V_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}} \times \cos \phi$$

or, $$P_{avg} = V I \cos \phi \text{ Watts} \qquad (7.57)$$

:-* **cos ϕ (*always,* – 1 < cos ϕ < + 1)** ***is the factor by which the power is being reduced in an A.C. circuit than that in case of a D.C. circuit **(*V I* Watts),** ***that is why*** **cos ϕ** ***is known as "Power Factor*** **(*P.F.*)*".***

If the circuit current I, shown in the phasor diagram in Fig. 7.41 (b), is resolved along the supply voltage ($I \cos \phi$) and along a perpendicular direction to supply voltage ($I \sin \phi$). The reader may observe from the equation (7.57) that:

i) *The component I cos ϕ is responsible for the power losses in the circuit, hence is known as **Watt-Full Current**.*

ii) On the other hand, *the component I sin ϕ is not responsible for any power loss in the circuit, hence is known as **Watt-Less Current**.*

Impedance Triangle: If we divide the voltage triangle [Fig. 7.41 (b)] by the circuit current I, we will get another similar triangle [Fig. 7.41 (c)] known as ***"Impedance Triangle"***. Refer to *impedance triangle* in Fig. 7.41 (c): (This *impedance triangle* along with *voltage triangle* and *power triangle* is very handy, tricky and useful for solving the numerical problems of single-phase A.C. circuits).

Clearly, $R = Z \cos \phi$, and, $X_C = Z \sin \phi$ [Always remember X_C is (–)ve] (7.58)

$$Z = (R - i\, X_C) \quad \text{and,} \quad Z = \sqrt{R^2 + X_C^2} \quad \text{and,} \quad \phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \cos^{-1}\left(\frac{R}{Z}\right) \qquad (7.59)$$

Power Triangle: If we multiply the voltage triangle [Fig. 7.41 (b)] by the circuit current I, we will get another similar triangle [Fig. 7.41 (d)] known as ***"Power Triangle"***. Refer to *power triangle* in Fig. 7.41 (d): (*This power triangle is very important. Also it is very handy, tricky and useful for solving the numerical problems of single-phase A.C. circuits*)

Apparent Power ($I^2 Z$ or $V I$, its unit is kVA): The hypotenuse of the power triangle is the apparent power used in the circuit. Its unit is Volt-Amp or Kilo-Volt-Amp.

$$\text{Apparent Power} = V\,I = I^2 Z \text{ (kVA)} \quad (7.60)$$

True Power ($I^2 R$ or $V\,I \cos\phi$, its unit is kW): The base of the power triangle is the power dissipated in the resistance and is true power as power dissipates in resistance only. Its unit is Watt or Kilo-Watt.

$$\text{True Power} = V\,I \cos\phi = I^2 R \text{ (kW)} \quad (7.61)$$

Reactive Power ($I^2 X_C$ or $V\,I \sin\phi$, its unit is kVAR): The perpendicular of the power triangle is the power stored and returned in the reactance and is known as reactive power as power is not dissipated in reactance. Its unit is Volt-Amp-Reactive or Kilo-Volt-Amp-Reactive.

$$\text{Reactive Power} = V\,I \sin\phi = I^2 X_C \text{ (kVAR)} \quad (7.62)$$

It can clearly be observed from above discussion and the power triangle, that:

$$(\text{Apparent Power})^2 = (\text{True Power})^2 + (\text{Reactive Power})^2$$

or, $$(\text{kVA})^2 = (\text{kW})^2 + (\text{kVAR})^2 \quad (7.63)$$

Problem 7.66: *Determine the magnitude and phase angle of a.c. current flowing through a circuit containing* $R = 10\ \Omega$ *and* $C = 50\ \mu F$ *connected in series across an a.c. source of* 200 *V*, 50 *Hz.*

[CBSE 1992-93]

Solution: $R = 10\ \Omega$, $C = 50\ \mu\text{F}$, $V = 200$ V, $f = 50$ Hz

The value of capacitive reactance offered by the capacitor to the flow of current may be given as:

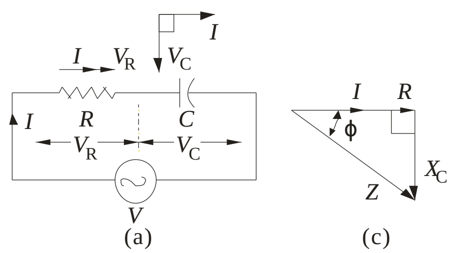


Fig. 7.42

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 50 \times 10^{-6}}$$

$$= 63.662\ \Omega$$

The value of the impedance of circuit, according to the impedance triangle shown in the Fig. 7.42 (b), may be given as:

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{(10)^2 + (63.662)^2} = 64.443\ \Omega$$

The value of a.c. current flowing through the circuit may be given as:

$$I = \frac{V}{Z} = \frac{200}{64.443} = 3.104 \text{ A}$$

The phase angle of the current flowing through the circuit, according to the phasor diagram drawn in the Fig. 7.42 (b), may be given as:

$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{63.662}{10}\right) = 81.07° \text{ (leading)}$$

Problem 7.67: *When an alternating voltage of* 220 *V is applied across a device X, a current of* 0.5 *A flows through the circuit which is in phase with the applied voltage. When the same voltage is applied across another device Y, the same current flows through the circuit but it leads the applied voltage by* (π / 2) *rad .Determine the: i) nature of device X and Y, ii) current flowing through the circuit, when same voltage is applied across the series combination of the devices X and Y.*

[CBSE 1996-97]

Solution: $V = 220$ V, $I_X = I_Y = 0.5$ A, $\phi_Y = (\pi / 2)$ rad (leading)

The current flowing through the device X is co-phasor with the applied voltage, so the device X is purely resistive in nature.

On the other hand, the current flowing through the device Y is leading the applied voltage by exactly 90°, so the device Y is purely capacitive in nature.

The resistance of the device X and the capacitive reactance of the device Y may be given as:

$$R = X_C = \frac{V}{I} = \frac{220}{0.5} = 440\ \Omega$$

The setup for both the devices (X and Y) connected in series across the supply source is shown in the Fig. 7.43 (a), and the corresponding phasor diagram is shown in the Fig. 7.43 (b).

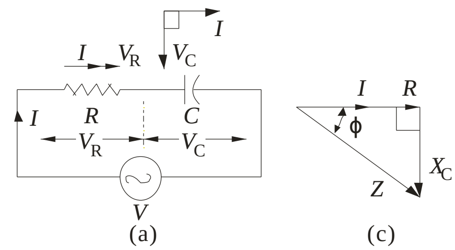


Fig. 7.43

The current flowing through the series combination of the devices X and Y may be given as:

$$I = \frac{V}{Z} = \frac{220}{\sqrt{(440)^2 + (440)^2}} = 0.3536\ \text{A}$$

Problem 7.68: *An alternating current of* 1.5 *mA* (*rms*) *and angular frequency* ω = 100 *rad/sec flows through a* 10 *k*Ω *resistor and* 0.50 μ*F capacitor connected in series. Determine the impedance of the circuit and the value of rms voltage across the capacitor.* [CBSE 1992-93]

Solution: I = 1.5 mA, ω = 100 rad/sec, R = 10 kΩ, C = 0.50 μF

The capacitive reactance offered by the capacitor to the flow of current may be given as:

$$X_C = \frac{1}{\omega C} = \frac{1}{100 \times 0.5 \times 10^{-6}}$$

$$= 20000\ \Omega = 20\ \text{k}\Omega$$

The impedance of the circuit, according to the impedance triangle shown in the Fig. 7.44, may be given as:

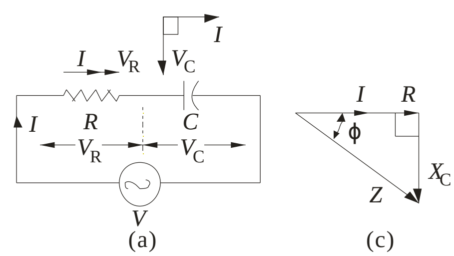


Fig. 7.44

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{(10)^2 + (20)^2}$$

$$= 22.361\ \text{k}\Omega$$

The rms value of the voltage across the capacitor may be given as:

$$V_C = I \times X_C = 1.5 \times 10^{-3} \times 20 \times 10^{3} = 30\ \text{V}$$

Problem 7.69: *A series circuit containing a resistor of* 20 Ω, *a capacitor C and an ammeter of negligible resistance is connected across a* 220 *V,* 50 *Hz supply source. If the reading of the ammeter is* 2.5 *A, determine the reactance and capacitance of the capacitor.* [Punjab 1998-99]

Solution: R = 20 Ω, V = 220 V, f = 50 Hz, I = 2.5 A

The expression for the current flowing through an R-C series circuit may be given as:

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + X_C^2}}$$

or, $$R^2 + X_C^2 = \left(\frac{V}{I}\right)^2$$

So, $X_C = \sqrt{\left(\frac{V}{I}\right)^2 - R^2} = \sqrt{\left(\frac{220}{2.5}\right)^2 - (20)^2} = 85.697\ \Omega$

or, $\frac{1}{2\pi f C} = 85.697$

So, $C = \frac{1}{2\pi f \times 85.697} = \frac{1}{2\pi \times 50 \times 85.697} = 37.144\ \mu\text{F}$

Problem 7.70: *A 5 W, 20 V lamp is to be operated on 200 V, 50 Hz a.c. mains. Determine the value of the capacitance required to run the lamp on the rated voltage and the rated current.*

Solution: Lamp = 5 W, 20 V, $V = 200$ V, $f = 50$ Hz

The required setup to operate the lamp on rated voltage and rated current, while it is connected across the given source, is shown in the Fig. 7.45 (a) and the corresponding phasor diagram is drawn in the Fig. 7.45 (b). Let the capacitor connected in series with the lamp is C.

The rated current of the lamp may be given as:

$$I_{rated} = \frac{P}{V} = \frac{5}{20} = 0.25\ \text{A}$$

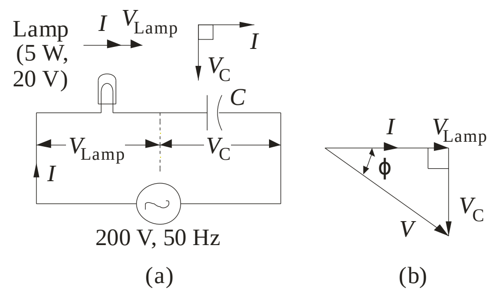


Fig. 7.45

The voltage across the capacitor, according to the phasor diagram drawn in the Fig. 7.45 (b) may be given as:

$$V_C = \sqrt{V^2 - V_{Lamp}^2} = \sqrt{(200)^2 - (20)^2}$$

$$= 198.998\ \text{V}$$

So, the value of the capacitive reactance required for the purpose may be given as:

$$X_C = \frac{V_C}{I_{rated}} = \frac{198.998}{0.25} = 795.992\ \Omega = \frac{1}{2\pi f C}$$

or, $C = \frac{1}{2\pi f \times 795.992} = \frac{1}{2\pi \times 50 \times 795.992} = 3.9989\ \mu\text{F} \approx 4\ \mu\text{F}$

Problem 7.71: *A resistor of 200 Ω and a capacitor of 15 μF are connected in series across a 220 V, 50 Hz a.c. source. Determine the: i) current flowing through the circuit, ii) voltage across the resistor and the capacitor. Is the algebraic sum of voltages across them is more than the source voltage? If yes, resolve the paradox.* [NCERT, CBSE 2007-08]

Solution: $R = 200\ \Omega$, $C = 15\ \mu$F, $V = 220$ V, $f = 50$ Hz

The capacitive reactance offered by the capacitor to the flow of current may be given as:

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 15 \times 10^{-6}} = 212.21\ \Omega$$

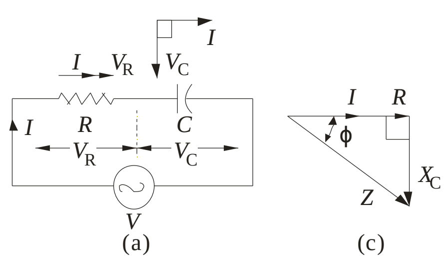


Fig. 7.46

The impedance of the circuit, according to the impedance triangle shown in the Fig. 7.46, may be given as:

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{(200)^2 + (212.21)^2} = 291.604\ \Omega$$

The current flowing through the circuit may be given as:

$$I = \frac{V}{Z} = \frac{220}{291.604} = 0.754 \text{ A}$$

The individual voltage across the resistor and the capacitor may respectively be given as:

$$V_R = I \times R = 0.754 \times 200 = 150.8 \text{ V}$$

$$V_C = I \times X_C = 0.754 \times 212.21 = 160.01 \text{ V}$$

So, the algebraic sum of the voltages across the resistor and the capacitor may be given as:

$$V_{algebric\ sum} = 150.8 + 160.01 = 310.81 > 220 \text{ V (the supply voltage)}$$

This is due to the fact that the supply voltage is vector sum of the individual voltages across the resistor and the capacitor.

Problem 7.72: *In an R-C series circuit;* $R = 30\ \Omega$, $C = 0.25\ \mu F$, $V = 100\ V$ *and* $\omega = 10{,}000$ *rad/sec. Determine the value of current through the circuit and the voltage across the individual circuit elements. Is the algebraic sum of voltages across them is more than the source voltage? If yes, resolve the paradox.* [CBSE 2003-04]

Solution: $R = 30\ \Omega$, $C = 0.25\ \mu\text{F}$, $V = 100$ V, $\omega = 10{,}000$ rad/sec

The capacitive reactance offered by the capacitor to the flow of current may be given as:

$$X_C = \frac{1}{\omega C} = \frac{1}{10000 \times 0.25 \times 10^{-6}} = 400\ \Omega$$

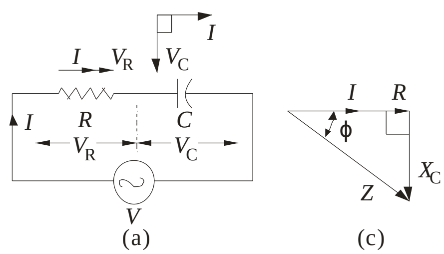


Fig. 7.47

The impedance of the circuit, according to the impedance triangle shown in the Fig. 7.47, may be given as:

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{(30)^2 + (400)^2} = 401.123\ \Omega$$

The current flowing through the circuit may be given as:

$$I = \frac{V}{Z} = \frac{100}{401.123} = 0.249 \text{ A}$$

The individual voltage across the resistor and the capacitor may respectively be given as:

$$V_R = I \times R = 0.249 \times 30 = 7.47 \text{ V}$$

$$V_C = I \times X_C = 0.249 \times 400 = 99.6 \text{ V}$$

So, the algebraic sum of the voltages across the resistor and the capacitor may be given as:

$$V_{algebric\ sum} = 7.47 + 99.6 = 107.07 > 100 \text{ V (the supply voltage)}$$

This is due to the fact that the supply voltage is vector sum of the individual voltages across the resistor and the capacitor.

Problem 7.73: *An a.c. circuit consists of a series combination of circuit elements X and Y. The current through the circuit is ahead of the voltage by an angle of* $(\pi / 4)$ *radians. If element X is a pure resistor of value* $100\ \Omega$. *Determine the: i) the nature of the circuit element Y, ii) rms value of the current flowing through the circuit, if rms value of the voltage is* 141 *V.* [CBSE 2003-04]

Solution: $\phi = \frac{\pi}{4}$ radians (leading), $R = 100\ \Omega$, $V = 141$ V

Since the current through the circuit is leading ahead of the applied voltage, so the element *Y* may be a pure capacitor or the combination of a capacitor and resistor. Let us assuming, for the time being, that the element *Y* is a pure capacitor.

The power factor, according to the impedance triangle shown in the Fig.7.48, may be given as:

$$\cos\phi = \frac{R}{Z}$$

So, $$Z = \frac{R}{\cos\phi} = \frac{100}{\cos 45°} = 100\sqrt{2} = 141.42\ \Omega$$

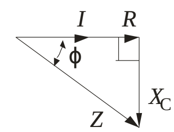


Fig. 7.48

So, the current flowing through the circuit may be given as:

$$I = \frac{V}{Z} = \frac{141}{141.42} = 0.997\ \text{A} \approx 1\ \text{A}$$

Problem 7.74: *A circuit containing a* 20 Ω *resistor and* 0.1 μ*F capacitor in series is connected across a* 230 *V a.c. supply of angular frequency* 100 *rad/sec. Determine the impedance of the circuit, the magnitude and the phase angle of the current flowing through the circuit.* [CBSE 1989-90]

Solution: $R = 20\ \Omega$, $C = 0.1\ \mu\text{F}$, $V = 230\ \text{V}$, $\omega = 100$ rad/sec

The value of the capacitive reactance offered by the capacitor to the flow of current may be given as:

$$X_C = \frac{1}{\omega C} = \frac{1}{100\times 0.1\times 10^{-6}} = 100{,}000\ \Omega = 100\ \text{k}\Omega$$

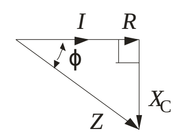


Fig. 7.49

The impedance of the circuit, according to the impedance triangle shown in the Fig. 7.49, may be given as:

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{(20)^2 + (100000)^2} = 10^5\ \Omega = 100\ \text{k}\Omega$$

The value of the current flowing through the circuit may be given as:

$$I = \frac{V}{Z} = \frac{230}{100\times 10^3} = 2.3\times 10^{-3}\ \text{A} = 2.3\ \text{mA}$$

The phase angle of the current flowing through the circuit may be given as:

$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{100000}{20}\right) = 89.989° \approx 90°$$

Problem 7.75: *A circuit consists of a resistor* 10 Ω *and a capacitor of* 0.1 μ*F in series. If an alternating emf of* 100 *V,* 50 *Hz is applied across the combination, determine the current flowing through the circuit.* [Haryana 1991-92]

Solution: $R = 10\ \Omega$, $C = 0.1\ \mu\text{F}$, $V = 100\ \text{V}$, $f = 50\ \text{Hz}$

The current flowing through the circuit may be given as:

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + X_C^2}} = \frac{V}{\sqrt{R^2 + \left(\frac{1}{2\pi f C}\right)^2}} = \frac{100}{\sqrt{(10)^2 + \left(\frac{1}{2\pi\times 50\times 0.1\times 10^{-6}}\right)^2}}$$

$$= 3.14\times 10^{-3}\ \text{A} = 3.14\ \text{mA}$$

Problem 7.76: *A* 20 *W,* 50 *V filament is connected in series with a capacitor to an a.c. mains of* 250 *V,* 50 *Hz. Determine the value of the capacitor required to operate the filament on correct voltage and the current.*

Solution: Filament = 20 W, 50 V, V = 250 V, f = 50 Hz

The rated current of the filament may be given as:

$$I_{rated} = \frac{P}{V} = \frac{20}{50} = 0.4 \text{ A}$$

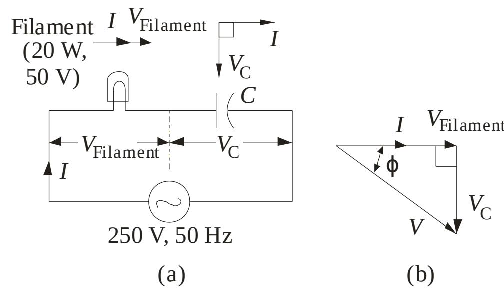


Fig. 7.50

The voltage across the capacitor, according to the phasor diagram drawn in the Fig. 7.50 (b), may be given as:

$$V_C = \sqrt{V^2 - V_{Filament}^2} = \sqrt{(250)^2 - (50)^2}$$

$$= 244.95 \text{ V}$$

The value of the capacitive reactance required may be given as:

$$X_C = \frac{V_C}{I_{rated}} = \frac{244.95}{0.4} = 612.35\ \Omega = \frac{1}{2\pi f C}$$

or, $$C = \frac{1}{2\pi f \times 612.35} = \frac{1}{2\pi \times 50 \times 612.35} = 5.198\ \mu\text{F}$$

Problem 7.77: *A* 1 µ*F capacitor is connected to* 220 *V*, 50 *Hz a.c. source. Determine the current through the capacitor. Also, determine the peak value of the voltage across the capacitor.*

Solution: C = 1 µF, V = 220 V, f = 50 Hz

The current through the capacitor may be given as:

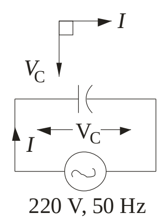


Fig. 7.51

$$I = \frac{V}{Z} = \frac{V}{X_C} = \frac{V}{(1/2\pi f C)} = V \times 2\pi f C$$

$$= 220 \times 2\pi \times 50 \times 1 \times 10^{-6} = 69.12 \text{ mA}$$

The voltage across the capacitor is same as that of the applied voltage, as the capacitor is the only element across the supply source. So, the peak value of the voltage across the capacitor may be given as:

$$V_0 = \sqrt{2} \times V_{rms} = \sqrt{2} \times 220 = 311.13 \text{ V}$$

Problem 7.78: *Determine the impedance of the circuit shown in the Fig.* 7.52, *for: i*) *direct current at steady state, ii*) *alternating current of frequency* (10 / π) *kHz.*

Solution: $R = 20\ \Omega$, $C = 2\ \mu$F, $f = \dfrac{10}{\pi}$ kHz

The capacitor behaves as open circuit for the d.c. currents at steady state $\left(as\ X_C \propto \frac{1}{f}\ and\ f = 0\ for\ d.c.\ currents\right)$.

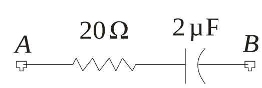


Fig. 7.52

So, the impedance of the circuit for d.c. currents may be given as:

$$Z_{d.c.\ current} = \infty$$

The impedance of the circuit for an a.c. current may be given as:

$$Z_{a.c.\ current} = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\frac{1}{2\pi f C}\right)^2}$$

$$= \sqrt{(20)^2 + \left(\frac{1}{2\pi \times (10/\pi) \times 10^3 \times 2 \times 10^{-6}}\right)^2} = 32.02\ \Omega$$

Problem 7.79: *A capacitor of reactance* 40 Ω *in series with a resistor of* 30 Ω *is connected to a.c. mains. Determine the phase difference between the supply voltage and the current flowing through the circuit.*

Solution: $X_C = 40\ \Omega$, $R = 30\ \Omega$

The phase difference between the supply voltage and the current flowing through the circuit, according to the phasor diagram drawn in the Fig. 7.53, may be given as:

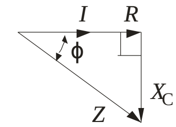


Fig. 7.53

$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{40}{30}\right) = 53.13° \text{ (leading)}$$

Problem 7.80: *A circuit has a resistance of* 100 Ω *and a capacitor in series. If the impedance of the circuit is* $100\sqrt{2}$ *Ω, determine the reactance of the circuit.*

Solution: $R = 100\ \Omega$, $Z = 100\sqrt{2}\ \Omega$

The capacitive reactance of the circuit, according to the impedance triangle shown in the Fig. 7.54, may be given as:

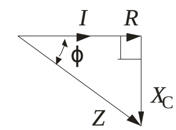


Fig. 7.54

$$X_C = \sqrt{Z^2 - R^2} = \sqrt{(100\sqrt{2})^2 - (100)^2} = 100\ \Omega$$

7.13 Resistance – Inductance – Capacitance (*R-L-C*) Series Circuit: Refer to the Fig. 7.55 (a), which shows an *R-L-C* series circuit in which a resistor, an inductor and a capacitor are connected in series carrying the same current *I*. So the current vector is same for all three elements [*note the direction of current vector drawn above the resistor, inductor and capacitor in Fig.* 7.55 (*a*) *is same*]. Voltage across the resistor (V_R) is co-phasor with the current vector [*see above the resistor in the Fig.* 7.55 (*a*)], voltage across the inductor (V_L) is 90° ahead of the current vector [*see above the inductor in the Fig.* 7.55 (*a*)] and voltage across the capacitor (V_C) is 90° behind the current vector [*see above the capacitor in the Fig.* 7.55 (*a*)]. Now, a phasor diagram of four voltages in the circuit V, V_R, V_L and V_C can be drawn as in Fig. 7.55 (b), known as **"*Voltage Triangle*"**. The vectors I and V_R are in phase, V_L is 90° ahead of I, V_C is 90° behind the current I and the resultant of V_R, V_L and V_C is V. Angle between the resultant voltage V and I is power factor angle, here I is lagging behind the supply voltage V by an angle ϕ. So, if supply voltage is given as:

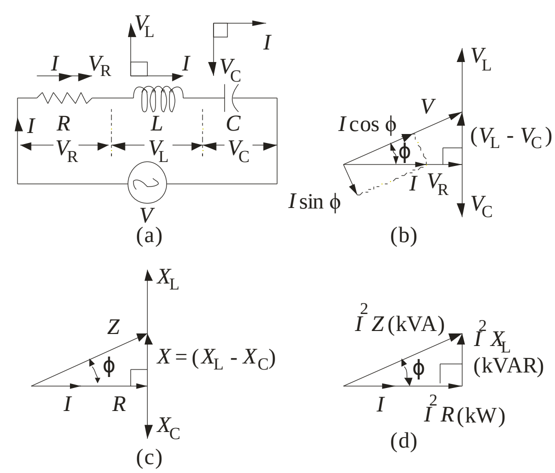


Fig. 7.55

$$v = V_0 \sin \omega t \tag{7.64}$$

the Circuit current may be given as:

$$i = I_0 \sin(\omega t - \phi) \tag{7.65}$$

Power Consumed in a Resistance – Inductance – Capacitance (*R*-*L*-*C*) Series Circuit: The instantaneous power in the circuit may be given by the product of instantaneous voltage and instantaneous current:

$$p = v\,i = V_0 \sin \omega t \times I_0 \sin (\omega t - \phi)$$

$$= \frac{V_0 I_0}{2} \times [\cos \{\omega t - (\omega t - \phi)\} - \cos \{\omega t + (\omega t - \phi)\}] = \frac{V_0 I_0}{2}\,[\cos \phi - \cos (2\omega t - \phi)]$$

Now, $$P_{avg} = \frac{1}{\pi} \times \int_0^{\pi} \left[\frac{V_0 I_0}{2}\{\cos \phi - \cos (2\omega t - \phi)\}\right] d(\omega t) = \frac{V_0 I_0}{2\pi} \times \left[(\omega t) \cos \phi - \frac{\sin (2\omega t - \phi)}{2}\right]_0^{\pi}$$

$$= \frac{V_0 I_0}{2\pi} \times \left[\left(\pi \cos \phi - \frac{\sin (2\pi - \phi)}{2}\right) - \left(0 \times \cos \phi - \frac{\sin (2 \times 0 - \phi)}{2}\right)\right]$$

$$= \frac{V_0 I_0}{2\pi} \times \left[\left(\pi \cos \phi - \frac{(-\sin \phi)}{2}\right) - \left(0 - \frac{(-\sin \phi)}{2}\right)\right]$$

$$= \frac{V_0 I_0}{2\pi} \times \left[\pi \cos \phi + \frac{\sin \phi}{2} - \frac{\sin \phi}{2}\right] = \frac{V_0 I_0}{2} \times \cos \phi = \frac{V_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}} \times \cos \phi$$

So, $$P_{avg} = V\,I \cos \phi \text{ Watts} \qquad (7.66)$$

:- cos ϕ (always, – 1 < cos ϕ < + 1) is the factor by which the power is being reduced in an A.C. circuit than that in case of a D.C. circuit (V I Watts), that is why cos ϕ is known as "Power Factor (P.F.)".

If the circuit current I, shown in the phasor diagram in Fig. 7.55 (b), is resolved along the supply voltage ($I \cos \phi$) and along a perpendicular direction to supply voltage ($I \sin \phi$). The reader may observe from the equation (7.66) that:

i) *The component I cos ϕ is responsible for the power losses in the circuit, hence is known as **Watt-Full Current**.*

ii) On the other hand, *the component I sin ϕ is not responsible for any power loss in the circuit, hence is known as **Watt-Less Current**.*

Impedance Triangle: If we divide the voltage triangle [Fig. 7.55 (b)] by the circuit current I, we will get another similar triangle [Fig. 7.55 (c)] known as ***impedance triangle***. Refer to impedance triangle in Fig. 7.55 (c): (*This impedance triangle along with voltage triangle and power triangle is very handy, tricky and useful for solving the numerical problems of single-phase AC circuits*)

Clearly, $R = Z \cos \phi$, and, $X = Z \sin \phi$ (7.67)

$Z = [R + i\,(X_L - X_C)]$ and, $Z = \sqrt{R^2 + (X_L - X_C)^2}$ (7.68)

$$\phi = \tan^{-1}\left[\frac{(X_L - X_C)}{R}\right] = \cos^{-1}\left(\frac{R}{Z}\right) \qquad (7.69)$$

Power Triangle: If we multiply the voltage triangle [Fig. 7.55 (b)] by the circuit current I, we will get another similar triangle [Fig. 7.55 (d)] known as ***power triangle***. Refer to power triangle in Fig. 7.55 (d): (*This power triangle is very important. Also it is very handy, tricky and useful for solving the numerical problems of single-phase AC circuits*).

Apparent Power ($I^2 Z$ or $V I$, its unit is kVA): The hypotenuse of the power triangle is the apparent power used in the circuit. Its unit is Volt-Amp or Kilo-Volt-Amp.

$$\text{Apparent Power} = V I = I^2 Z \text{ (kVA)} \tag{7.70}$$

True Power ($I^2 R$ or $V I \cos \phi$, its unit is kW): The base of the power triangle is the power dissipated in the resistance and is true power as power dissipates in resistance only. Its unit is Watt or Kilo-Watt.

$$\text{True Power} = V I \cos \phi = I^2 R \text{ (kW)} \tag{7.71}$$

Reactive Power ($I^2 X$ or $V I \sin \phi$, its unit is kVAR): The perpendicular of the power triangle is the power stored and returned in the reactance and is known as reactive power as power is not dissipated in reactance. Its unit is Volt-Amp-Reactive or Kilo-Volt-Amp-Reactive.

$$\text{Reactive Power} = V I \sin \phi = I^2 X \text{ (kVAR)} \tag{7.72}$$

It can clearly be observed from above discussion and the power triangle, that:

$$(\text{Apparent Power})^2 = (\text{True Power})^2 + (\text{Reactive Power})^2$$

or, $$(\text{kVA})^2 = (\text{kW})^2 + (\text{kVAR})^2 \tag{7.73}$$

Susceptance: The reciprocal of the reactance of an a.c. circuit is known as *susceptance*. Its SI unit is Siemens (S) or Ohm^{-1}.

Admittance: The reciprocal of the impedance of an a.c. circuit is known as *admittance*. Its SI unit is Siemens (S) or Ohm^{-1}.

7.14 Resonance in Series *R-L-C* Circuit (Voltage Resonance / Selector Circuit): ***If the applied voltage (V) and the circuit current (I) becomes co-phasor in an R-L-C (Series / Parallel) circuit, the condition is said to be the resonance condition in the R-L-C (Series / Parallel) circuit.***

Consider the Fig. 7.56, which shows a series *R-L-C* circuit along with its voltage triangle and impedance triangle. The impedance of a series *R-L-C* circuit may be given as:

$$\boldsymbol{Z} = \sqrt{R^2 + (X_L - X_C)^2} \tag{7.74}$$

If this circuit has, $X_L = X_C$, *i.e.* $2\pi f_0 L = \dfrac{1}{2\pi f_0 C}$ for certain value of frequency, f_0.

Then, $Z = R$, and the circuit current (I) becomes co-phasor with the applied voltage (V), as $V_L = I X_L$ and $V_C = I X_C$ will cancel out each other.

This is the required condition for series or voltage resonance.

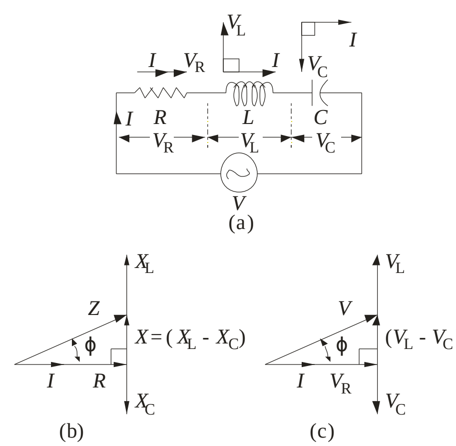


Fig. 7.56 (*R-L-C* Series resonance)
($V_L = V_C$ and $X_L = X_C$)

So, if, $X_L = X_C$

or, $$2\pi f_0 L = \frac{1}{2\pi f_0 C}$$

or, $$f_0 = \frac{1}{2\pi\sqrt{LC}}, \text{ where } f_0 = f_r = \text{resonance frequency.} \tag{7.75}$$

Here are few important points about series *R-L-C* (Voltage) resonance / selector circuit:

i) As, $X_L = X_C$ at resonance, $X_L - X_C = 0$, hence net reactance of the circuit is zero at resonance.

ii) Impedance of the circuit $Z = R$ at the resonance and this is the *minimum value of impedance for this circuit*.

iii) Power factor of the load and hence of the circuit is unity.

iv) The current flowing through the circuit $\left(I_0 = \frac{V}{Z} = \frac{V}{R}\right)$ at resonance is *maximum value of current* for this circuit.

v) The power supplied by the source, $P = V I_0$, as p.f. ($\cos \phi$) of the load / circuit is unity.

Current -vs- Frequency Curve: Fig 7.57 shows the graph of current drawn for a series *R-L-C* circuit against frequency of the applied voltage. The current attains the maximum value only at unity power factor, *i.e.* resonance frequency (f_0). The current decreases on either side of this unity power factor point (resonance frequency, f_0).

Half Power Frequencies: *Half power frequencies are the frequencies* (f_1 *and* f_2) *at which power dissipated in the circuit is half of that the power dissipated at the resonance frequency* (*the maximum power dissipates in the circuit at the resonance frequency*, f_0).

Power dissipated in the circuit at resonance frequency, $P = I_0^2 R$

If the current corresponding to half of the power dissipated at resonance frequency is say I', the power dissipated at half power frequencies may be given by:

$$P_{\left(\frac{1}{2}\right)} = (I')^2 \times R = \frac{P}{2} = \frac{I_0^2 R}{2} = \left(\frac{I_0}{\sqrt{2}}\right)^2 \times R$$

So, $$I' = \frac{I_0}{\sqrt{2}} \qquad (7.76)$$

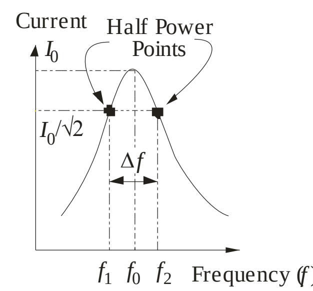


Fig. 7.57

The line corresponding to the current $\frac{I_0}{\sqrt{2}}$ on the graph Current –vs– frequency intersects it at frequencies f_1 and f_2, hence these two frequencies are known as ***half power frequencies***.

And, impedance of the circuit corresponding to this current $\left(I' = \frac{I_0}{\sqrt{2}}\right)$ at half power frequencies may be given as:

$$Z_{\left(\frac{1}{2}\right)} = \frac{V}{I'} = \frac{V}{(I_0/\sqrt{2})} = \sqrt{2} \times \frac{V}{I_0} = \sqrt{2} \times R \qquad \left(\text{As } \frac{V}{I_0} = R, \text{ at resonance frequency}\right)$$

Significance of Half Power Frequencies: These half power frequencies has a special significance: **"*Any signal, having the near-by frequency for which circuit is resonating, has sufficient power within these half power frequencies and can be analyzed or listened* (*e.g. in case of radio-sets, mobile phones and televisions*) *after proper amplification for information hidden in the signal***.

R-L-C* Series Circuit as Selector / Acceptor Circuit:** There are several radio signals in the air, *e.g.* various radio stations, various television channels, mobile communication, internet signals etc. at different frequencies [various separate frequency bands (known as *band width*) are allotted to them, the reader must be heard about 2G-spectrum scam], which are to be separated from other frequencies / signals for retrieving the information hidden inside them. So, *a circuit is designed to resonate at the frequency to which we are trying to select*. Now the current, for the signal having resonance frequency, in the circuit attains the maximum value (while all other signals have very less current in the circuit at this frequency) and has sufficient strength in between the half power frequencies to be amplified and analysis of the information hidden in it. So, *the voltage or series resonance circuit is also known as* ***selector circuit.

As discussed earlier also, the impedance of the circuit at half power frequencies may be given as,

$$Z_{\left(\frac{1}{2}\right)} = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{2} \times R \tag{7.77}$$

or, $R^2 + (X_L - X_C)^2 = 2R^2$ or, $(X_L - X_C)^2 = R^2$

So, $(X_L - X_C)^2 = \left(\omega L - \dfrac{1}{\omega C}\right)^2 = \pm R^2$

or,
$$\left(\omega_1 L - \frac{1}{\omega_1 C}\right) = -R \tag{7.78}$$

and,
$$\left(\omega_2 L - \frac{1}{\omega_2 C}\right) = +R \tag{7.79}$$

Equation (7.78) + (7.79):

$$(\omega_1 + \omega_2) \times L - \frac{1}{C} \times \left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right) = 0$$

or, $(\omega_1 + \omega_2) \times L - \dfrac{1}{C} \times \left(\dfrac{\omega_1 \omega_2}{\omega_1 + \omega_2}\right) = 0$

or, $L - \dfrac{1}{C} \times \dfrac{1}{\omega_1 \omega_2} = 0$

or,
$$\frac{1}{\omega_1 \omega_2} = LC \quad \text{or,} \quad \frac{1}{\sqrt{LC}} = \omega_0 = \sqrt{\omega_1 \omega_2} \tag{7.80}$$

Equation (7.79) – (7.78):

$$(\omega_2 - \omega_1) \times L + \frac{1}{C} \times \left(\frac{1}{\omega_1} - \frac{1}{\omega_2}\right) = 2R$$

or, $(\omega_2 - \omega_1) \times L + \dfrac{1}{C} \times \left(\dfrac{\omega_2 - \omega_1}{\omega_1 \omega_2}\right) = (\omega_2 - \omega_1) \times \left(L + \dfrac{1}{C} \times \dfrac{1}{\omega_1 \omega_2}\right) = 2R$

or, $(\omega_2 - \omega_1) \times \left(L + \dfrac{LC}{C}\right) = (\omega_2 - \omega_1) \times (L + L) = 2L \times (\omega_2 - \omega_1) = 2R$

or, $(\omega_2 - \omega_1) = \dfrac{R}{L}$

or,
$$(f_2 - f_1) = \frac{R}{2\pi L} = \text{Band Width } (\Delta f) \tag{7.81}$$

Now, It can easily be interpreted from the current –vs– frequency graph that:

Half of the Band Width, $\dfrac{\Delta f}{2} = f_2 - f_0 = f_0 - f_1 = \dfrac{R}{2\pi L} \times \dfrac{1}{2} = \dfrac{R}{4\pi L}$

So, half power frequencies may now be given by:

$$f_2 = f_0 + \frac{R}{4\pi L} \qquad \text{and,} \qquad f_1 = f_0 - \frac{R}{4\pi L}$$

or, in general we may write it as:

$$f_{(Half\,Power)} = f_0 \pm \frac{R}{4\pi L} \tag{7.82}$$

Quality Factor of Series (Voltage) Resonance Circuit: We have seen that in the series *R-L-C* resonance circuit the voltage across inductor and the voltage across the capacitor cancel out each other at the time of resonance. So, the supply voltage appears across resistance, *i.e.* $V_L = V_C$ and $V = V_R$. So, we can increase the voltage across inductor and capacitor to any finite value choosing appropriate values of inductance and capacitance keeping the condition of series resonance $\left(f_0 = \dfrac{1}{2\pi\sqrt{LC}}\right)$ satisfied with-out any damage to the circuit. ***"So, series resonance (circuit) is also known as voltage resonance (circuit), as voltage amplification occurs during series resonance"***.

"The ratio (amount) of voltage amplification in the series resonance circuit is known as quality factor of the series resonant circuit". The high quality factor indicates the high sharpness of the resonance, as the current curve has a sharper peak for an *R-L-C* series circuit having high quality factor.

Quality factor for a series *R-L-C* circuit may be given as:

$$Q = \frac{V_L}{V} = \frac{I\,X_L}{I\,R} = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{2\pi f_0 L}{R}$$

$$= f_0 \times \frac{2\pi L}{R} = \frac{1}{2\pi\sqrt{LC}} \times \frac{2\pi L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} \tag{7.83}$$

Also, $\quad Q = f_0 \times \dfrac{2\pi L}{R} = \dfrac{f_0}{(R/2\pi L)} = \dfrac{f_0}{\Delta f} = \dfrac{\text{Resonance Frequency}}{\text{Band Width}}$

So, $$Q = \frac{\omega L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{f_0}{\Delta f} \tag{7.84}$$

Another definition of quality factor: *Ratio of resonance frequency* (f_0) *to bandwidth* (Δf) *is known as the quality factor of series* (*Voltage*) *resonance circuit*.

$$Q = \frac{\text{Resonance Frequency}}{\text{B.W.}} = \frac{f_0}{\Delta f} = \frac{1}{2\pi\sqrt{LC}} \times \frac{2\pi L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} \tag{7.85}$$

The quality factor of the circuit may be increased by decreasing the band width (Δf) of the circuit. So, narrow bandwidth indicates more sharpness at the resonance.

Selectivity of R-L-C series Circuit: It is directly proportional to the quality factor of the circuit, the high quality factor means the higher selectivity of the circuit. The selectivity of the series resonance circuit may be defined as, "*The ability to discriminate a particular signal of certain frequency from various mixed signals of many frequencies*".

Problem 7.81: *Determine the impedance of an R-L-C series circuit having R = 40 Ω, X_L = 220 Ω and X_C = 250 Ω.* [CBSE 1993-94]

Solution: $R = 40\ \Omega$, $X_L = 220\ \Omega$, $X_C = 250\ \Omega$

The impedance triangle for the given *R-L-C* circuit is drawn in the Fig. 7.58. Now, the impedance of the circuit, according to the impedance triangle, may be given as:

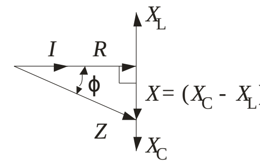


Fig. 7.58

$$Z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{(40)^2 + (250 - 220)^2} = 50\ \Omega$$

Problem 7.82: *Determine the readings of the voltmeter and ammeter shown in the circuit diagram given in the Fig. 7.59.*

Solution: $R = 45\ \Omega$, $X_L = X_C = 4\ \Omega$, $V = 90$ V

The reader may easily observe that, the inductive and the capacitive reactance are equal, *i.e.* $X_L = X_C$ in the given circuit. So, the circuit is resonating at the given frequency. Hence, the two voltages across the inductor ($V_L = I\ X_L$) and that across the capacitor ($V_C = I\ X_C$) are equal in magnitude.

So, the reading of the voltmeter, according to the phasor diagram drawn in the Fig. 7.59 (b), may be given as:

$$V = V_L - V_C = 0$$

So, the impedance of the circuit may be given as:

$$Z = R$$

and, the current through the circuit (reading of the ammeter) may be given as:

$$I = \frac{V}{Z} = \frac{V}{R} = \frac{90}{45} = 2\ \text{A}$$

The reader may cross check that the reading of the voltmeter may also be given as:

$$V = (V_L - V_C) = I \times (X_L - X_C) = 2 \times (4 - 4) = 0$$

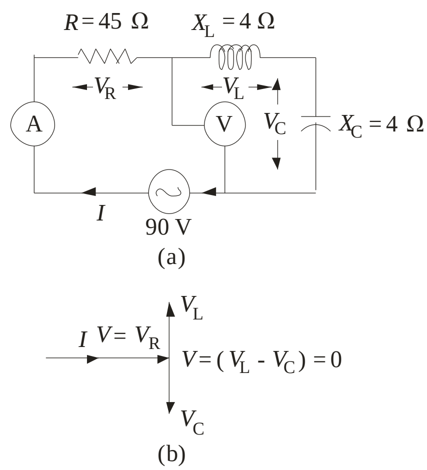


Fig. 7.59

Problem 7.83: *A 0.3 H inductor, a 60 μF capacitor and a 50 Ω resistor are connected in series with a 120 V, 60 Hz supply. Determine the: i) impedance of the circuit, ii) magnitude and phase angle of the current flowing through the circuit.* [CBSE 1993-94]

Solution: $L = 0.3$ H, $C = 60\ \mu$F, $R = 50\ \Omega$, $V = 120$ V, $f = 60$ Hz

The reactance offered by the circuit to the flow of the current may be given as:

$$X = X_L - X_C = 2\ \pi\ f\ L - \frac{1}{2\pi f C} = 2\ \pi \times 60 \times 0.3 - \frac{1}{2\pi \times 60 \times 60 \times 10^{-6}}$$

$$= 113.097 - 44.21 = 68.887\ \Omega$$

Now, the impedance of the circuit may be given as:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

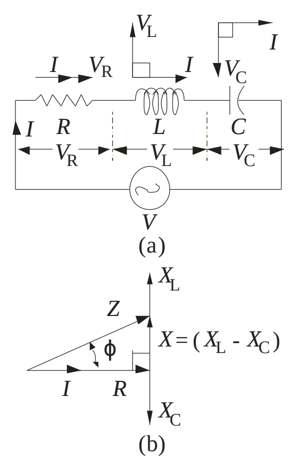


Fig. 7.60

$$= \sqrt{(50)^2 + (68.887)^2} = 85.12\ \Omega$$

So, the magnitude of the current flowing through the circuit may be given as:

$$I = \frac{V}{Z} = \frac{120}{85.12} = 1.41\ \text{A}$$

The phase angle of the current flowing through the circuit, according to the phasor diagram drawn in the Fig. 7.60 (b), may be given as:

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{68.887}{50}\right) = 54.03°$$

Problem 7.84: *A resistor of* 50 Ω, *an inductor of* (20 / π) *H, and a capacitor of* (5 / π) μ*F are connected in series across a* 230 *V,* 50 *Hz a.c. source. Determine the impedance of the circuit and the magnitude and phase angle of the current flowing through the circuit.* [CBSE 1985-86]

Solution: $R = 50\ \Omega$, $L = \frac{20}{\pi}$ H, $C = \frac{5}{\pi}$ μF, $V = 230$ V, $f = 50$ Hz

The impedance triangle for the given *R-L-C* circuit is drawn in the Fig. 7.61. Now, the impedance of the circuit, according to the impedance triangle, may be given as:

$$Z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{R^2 + \left(2\pi f L - \frac{1}{2\pi f C}\right)^2}$$

$$= \sqrt{(50)^2 + \left(2\pi \times 50 \times \frac{20}{\pi} - \frac{1}{2\pi \times 50 \times (5/\pi) \times 10^{-6}}\right)^2}$$

$$= \sqrt{(50)^2 + (2000 - 2000)^2} = 50\ \Omega$$

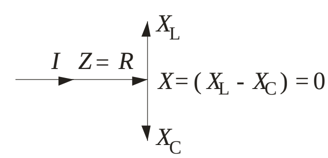


Fig. 7.61

Since, $X_L = X_C$ for the given circuit, so the circuit is resonating and the frequency 50 Hz is resonance frequency for the given circuit.

Problem 7.85: *A* 100 *mH inductor, a* 20 μ*F capacitor and a* 10 Ω *resistor are connected in series across a* 100 *V,* 50 *Hz a.c. source. Determine the: i*) *resonance frequency, ii*) *impedance of the circuit at resonance, iii*) *current at resonance.*

Solution: $L = 100$ mH, $C = 20$ μF, $R = 10\ \Omega$, $V = 100$ V, $f = 50$ Hz

The resonance frequency for an *R-L-C* series circuit may be given as:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{100 \times 10^{-3} \times 20 \times 10^{-6}}} = 112.54\ \text{Hz}$$

The impedance of the circuit at resonance ($X_L = X_C$, or $X_L - X_C = 0$) may be given as:

$$Z = R = 10\ \Omega$$

The current flowing through the circuit at resonance may be given as:

$$I = \frac{V}{Z} = \frac{V}{R} = \frac{100}{10} = 10\ \text{A}$$

Problem 7.86: *A resistor of* 12 Ω, *a capacitor of* 227.36 μ*F and a pure inductor of* 100 *mH are connected in series and placed across a* 200 *V,* 50 *Hz a.c. supply. Determine: i*) *the current flowing through* *the circuit, ii) the phase angle between the applied voltage and the current flowing through the circuit.*

Solution: $R = 12\ \Omega$, $C = 227.36\ \mu F$, $L = 0.1$ H, $V = 200$ V, $f = 50$ Hz

The reactance offered by the circuit to the flow of the current may be given as:

$$X = X_L - X_C = 2\pi f L - \frac{1}{2\pi f C}$$

$$= 2\pi \times 50 \times 0.1 - \frac{1}{2\pi \times 50 \times 227.36 \times 10^{-6}}$$

$$= 31.416 - 14 = 17.416\ \Omega \text{ (inductive)}$$

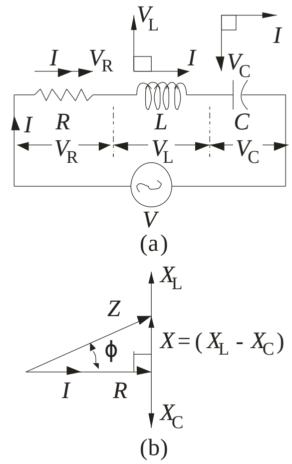


Fig. 7.62

Now, the impedance of the circuit may be given as:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(12)^2 + (17.416)^2}$$

$$= 21.15\ \Omega$$

So, the magnitude of the current flowing through the circuit may be given as:

$$I = \frac{V}{Z} = \frac{200}{21.15} = 9.46 \text{ A}$$

The phase angle of the current flowing through the circuit, according to the phasor diagram drawn in the Fig. 7.62 (b), may be given as:

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{17.416}{12}\right) = 55.43° \text{ (lagging)}$$

Problem 7.87: *An R-L-C series circuit is consisting of a resistance of* 10 Ω, *an inductor of unknown inductance and a capacitor of* 26.526 μ*F. This combination is found to be resonating, when connected across a* 300 *V,* 100 *Hz a.c. source. Determine: i) the value of inductance connected in the circuit, ii) the current in the circuit at resonance.* [CBSE 1993-94]

Solution: $R = 10\ \Omega$, $C = 26.526\ \mu F$, $V = 300$ V, $f = 100$ Hz

The expression for the resonance frequency of an *R-L-C* series circuit may be given as:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

or, $$L = \frac{1}{(2\pi f)^2 \times C} = \frac{1}{(2\pi \times 100)^2 \times 26.526 \times 10^{-6}} = 95.49 \text{ mH}$$

The impedance of the circuit at resonance ($X_L = X_C$, or $X_L - X_C = 0$) may be given as:

$$Z = R = 10\ \Omega$$

So, the current flowing through the circuit at resonance may be given as:

$$I = \frac{V}{Z} = \frac{V}{R} = \frac{300}{10} = 30 \text{ A}$$

Problem 7.88: *An inductor coil connected to a* 6 *V battery draws a steady current of* 12 *A from the battery. The coil is then connected in series with a capacitor and then connected across a* 6 *V a.c. source. If* *the current through the circuit is co-phasor with the applied voltage, determine the rms value of the current.*

Solution: $V_{d.c.} = 6$ V, $I_{d.c.} = 12$ A, $V_{a.c.} = 6$ V, $\phi = 0$

When an inductive coil is connected across a d.c. source, the only active component of the coil at steady state is the resistance of the coil as inductor behaves as short circuit for a d.c. source. So, the resistance of the coil may be given as:

$$R = \frac{V_{d.c.}}{I_{d.c.}} = \frac{6}{12} = 0.5\ \Omega$$

If the phase angle between the applied a.c. voltage and the circuit current is zero, it indicates that the circuit is resonating at this particular frequency. The impedance of an a.c. electrical circuit at resonance may be given as:

$$Z = R = 0.5\ \Omega$$

So, the current through the a.c. circuit may be given as:

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{R} = \frac{6}{0.5} = 12 \text{ A } (\textit{i.e.} \text{ same as that in case of its d.c. operation})$$

Problem 7.89: *A radio wave of wavelength* 300 *m can be transmitted by a transmission center. A condenser of capacity* 2.4 μ*F is available. Determine the inductance of required coil for analysis of the transmitted signal.*

Solution: $\lambda = 300$ m, $C = 2.4$ μF

The frequency of the transmitted signal may be given as:

$$f = \frac{c}{\lambda} = \frac{3\times10^8}{300} = 1 \text{ MHz}$$

If the resonance frequency of the designed circuit will be equal to the frequency of the transmitted signal, it can be analyzed easily after amplification.

So, $$f_0 = \frac{1}{2\pi\sqrt{LC}} = 1 \times 10^6$$

or, $$L = \frac{1}{(2\pi f)^2 \times C} = \frac{1}{(2\pi\times1\times10^6)^2 \times 2.4\times10^{-6}} = 1.055 \times 10^{-8} \text{ H} = 10.55 \text{ nH}$$

Problem 7.90: *A* 25 μ*F capacitor, a* 100 *mH inductor and a* 25 Ω *resistor are connected in series across an a.c. source of emf e* = 310 *sin* 314*t Volts. Determine: i*) *frequency of the source, ii*) *reactance of the circuit, iii*) *impedance of the circuit, iv*) *magnitude of the current flowing through the circuit, v*) *phase angle of the current flowing through the circuit, vi*) *expression for the instantaneous value of the current flowing through the circuit, vii*) *individual voltages across the resistor, inductor and capacitor, viii*) *draw a phasor diagram for all these quantities, ix*) *the value of inductance which will make the resultant impedance of the circuit to be minimum.*

Solution: $C = 25$ μF, $L = 0.1$ H, $R = 25\ \Omega$, $e = 310 \sin 314t$ Volts

Comparing the expression for the given emf with the standard emf expression:

$$E_0 = 310 \text{ V}, \qquad \omega = 314 \text{ rad/sec}$$

So, $$f = \frac{\omega}{2\pi} = \frac{314}{2\pi} = 49.975 \text{ Hz} \approx 50 \text{ Hz}$$

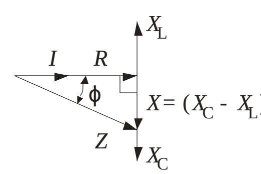


Fig. 7.63

The inductive reactance and capacitive reactance in the circuit may respectively be given as:

$$X_L = \omega L = 314 \times 0.1 = 31.4\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{314 \times 25 \times 10^{-6}} = 127.39\ \Omega$$

So, the net reactance of the circuit may be given as:

$$X = |X_L - X_C| = |31.4 - 127.39| = 95.99\ \Omega \text{ (capacitive)}$$

The net impedance of the circuit may be given as:

$$Z = \sqrt{R^2 + X^2} = \sqrt{(25)^2 + (95.99)^2} = 99.19\ \Omega$$

The current flowing through the circuit may be given as:

$$I = \frac{E_{rms}}{Z} = \frac{E_0}{\sqrt{2} \times Z} = \frac{310}{\sqrt{2} \times 99.19} = 2.21 \text{ A}$$

The phase angle of the current may be given as:

$$\phi = \tan^{-1}\left(\frac{X}{R}\right) = \tan^{-1}\left(\frac{95.99}{25}\right) = 75.4° \text{ (leading)} = 75.4° \times \frac{\pi}{180°} = 1.32 \text{ rad (leading)}$$

The expression for the instantaneous value of the current flowing through the circuit may be given as:

$$i = I_0 \sin(314\, t + 1.31) = \sqrt{2} \times I_{rms} \sin(314\, t + 1.31)$$

$$= \sqrt{2} \times 2.21 \sin(314\, t + 1.31) = 3.125 \sin(314\, t + 1.31) \text{ Amp}$$

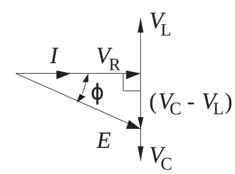


Fig. 7.64

The voltage across individual circuit elements may be given as:

$$V_R = I \times R = 2.21 \times 25 = 55.25 \text{ V}$$

$$V_L = I \times X_L = 2.21 \times 31.4 = 69.394 \text{ V}$$

$$V_C = I \times X_C = 2.21 \times 127.39 = 281.532 \text{ V}$$

The phasor diagram for the various voltages and current flowing through the circuit is drawn in the Fig. 7.64.

The minimum impedance of the circuit implies the resonance in the circuit. So, the value of required inductance may be given by the expression:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

or, $$L = \frac{1}{(2\pi f)^2 \times C} = \frac{1}{(314)^2 \times 25 \times 10^{-6}} = 0.40569 \text{ H} = 405.69 \text{ mH}$$

Problem 7.91: *A* 2 μ*F capacitor, a* 100 Ω *resistor and an 8 H inductor are connected in series across an a.c. source of emf having a peak value of* 200 *V, and the current drawn by the circuit is maximum current. Determine for maximum current: i*) *the inductive and capacitive reactance for the circuit, ii*) *total impedance of the circuit, iii*) *peak value of current in the circuit, iv*) *phase difference between voltage across the inductor and resistor, v*) *phase difference between voltage across the inductor and capacitor.* [ISCE 1997-98]

Solution: $C = 2\ \mu\text{F}$, $R = 100\ \Omega$, $L = 8\text{ H}$, $E_0 = 200\text{ V}$

Since, the current drawn by this circuit is maximum current from the a.c. source, it indicates that the circuit is resonating. So the frequency of the source is resonance frequency of the circuit which may be given as:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{8\times 2\times 10^{-6}}} = 39.79 \text{ Hz}$$

So, the inductive and capacitive reactance (although equal in magnitude) in the circuit may respectively be given as:

$$X_L = 2\,\pi\,f\,L = 2\,\pi \times 39.79 \times 8 = 2000\ \Omega$$

And, $$X_C = \frac{1}{2\pi\,f\,C} = \frac{1}{2\pi\times 39.79\times 2\times 10^{-6}} = 2000\ \Omega$$

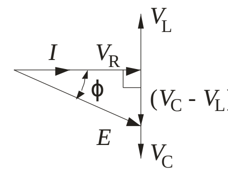


Fig. 7.65

The peak value of the current flowing through the circuit (at resonance) may be given as:

$$I_0 = \frac{E_0}{Z} = \frac{E_0}{R} = \frac{200}{100} = 2 \text{ A}$$

The phase difference between the voltages across inductor and resistor may be given as:

$$\phi_{RL} = 90° \text{ (voltage across the inductor leading)}$$

The phase difference between the voltages across inductor and capacitor may be given as:

$$\phi_{LC} = 180° \text{ (voltage across the inductor leading)}$$

Problem 7.92: *In a series R-L-C circuit the resonance frequency is* 800 *Hz. The half power frequencies are obtained at frequencies* 745 *Hz and* 855 *Hz. Determine the Q-factor of the circuit. Also determine the bandwidth of the circuit.*

Solution: $f_0 = 800$ Hz, $f_1 = 745$ Hz, $f_2 = 855$ Hz

The band width of the circuit may be given as:

$$\text{B.W.} = \Delta f = f_2 - f_1 = 855 - 745 = 110 \text{ Hz}$$

The quality factor of the circuit may be given as:

$$Q = \frac{f_0}{\Delta f} = \frac{800}{110} = 7.273$$

Problem 7.93: *A resistor of* 40 Ω, *an inductor of* 3 *mH and a capacitor of* 2 μ*F are connected in series across a* 110 *V,* 5000 *Hz a.c. source. Determine the value of current flowing through the circuit.*

Solution: $R = 40\ \Omega$, $L = 3$ mH, $C = 2\ \mu$F, $V = 110$ V, $f = 5000$ Hz

The impedance of the circuit may be given as:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(2\pi\,f\,L - \frac{1}{2\pi\,f\,C}\right)^2}$$

or, $$Z = \sqrt{(40)^2 + \left(2\pi\times 5000\times 3\times 10^{-3} - \frac{1}{2\pi\times 5000\times 2\times 10^{-6}}\right)^2}$$

$$= \sqrt{(40)^2 + (94.248 - 15.916)^2} = 87.954\ \Omega$$

The current flowing through the circuit may be given as:

$$I = \frac{V}{Z} = \frac{110}{87.954} = 1.25 \text{ A}$$

Problem 7.94: *A resistor of* 10 Ω, *an inductor of* 200 *mH and a capacitor C are connected in series across a* 100 *V,* 50 *Hz a.c. source. If the applied voltage and the current flowing through the circuit are co-phasor, determine the capacitance C.* [CBSE 2005-06]

Solution: $R = 10\ \Omega$, $L = 200$ mH, $V = 100$ V, $f = 50$ Hz, $\phi = 0°$

Since, the applied voltage and the current flowing through the circuit are co-phasor, so the circuit is resonating for the given frequency and the value of the capacitance C may be given by the relationship:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

or, $$C = \frac{1}{(2\pi f)^2 \times L} = \frac{1}{(2\pi \times 50)^2 \times 200 \times 10^{-3}} = 50.66\ \mu\text{F}$$

Problem 7.95: *A* 48 Ω *resistor, a* 50 *mH inductor and a* 50 μ*F capacitor are connected in series across an a.c. source of emf e* = 310 *sin* 314*t Volts. Determine the net reactance of the circuit and tell its nature. Also determine the phase angle between the applied voltage and the current flowing through the circuit.*

Solution: $R = 48\ \Omega$, $L = 50$ mH, $C = 50\ \mu$F, $e = 310 \sin 314t$ Volts

Comparing the expression for the emf with standard emf expression:

$E_0 = 310$ V, $\omega = 314$ rad/sec

The inductive reactance and the capacitive reactance in the circuit may respectively be given as:

$$X_L = \omega L = 314 \times 50 \times 10^{-3} = 15.7\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{314 \times 50 \times 10^{-6}} = 63.694\ \Omega$$

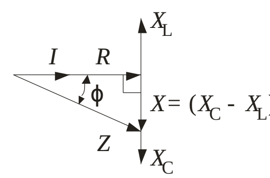


Fig. 7.66

So, the net reactance of the circuit may be given as:

$$X = X_C - X_L = 63.694 - 15.7 = 47.994\ \Omega \text{ (capacitive)}$$

The phase angle between the applied voltage and the current flowing through the circuit, according to the phasor diagram drawn in the Fig. 7.66, may respectively be given as:

$$\phi = \tan^{-1}\left(\frac{X}{R}\right) = \tan^{-1}\left(\frac{47.994}{48}\right) \approx 45° \text{ (leading)}$$

$$I_{rms} = \frac{E_{rms}}{Z} = \frac{E_0}{\sqrt{2}Z} = \frac{E_0}{\sqrt{2} \times \sqrt{R^2 + X^2}} = \frac{310}{\sqrt{2} \times \sqrt{(48)^2 + (47.994)^2}} = 3.229 \text{ A}$$

Problem 7.96: *An R-L-C series circuit with R* = 120 Ω, *L* = 100 *mH and C* = 100 μ*F is connected across an a.c. source of emf e* = 30 *sin* 100*t Volts. Determine the impedance, peak value of the current flowing through the circuit and the resonance frequency for the circuit.*

Solution: $R = 120\ \Omega$, $L = 100$ mH, $C = 100\ \mu$F, $e = 30 \sin 100t$ Volts

Comparing the expression for the emf with standard emf expression:

$E_0 = 30$ V, $\omega = 100$ rad/sec

The inductive reactance and the capacitive reactance in the circuit may respectively be given as:

$$X_L = \omega L = 100 \times 100 \times 10^{-3} = 10\ \Omega$$

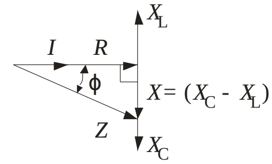


Fig. 7.67

$$X_C = \frac{1}{\omega C} = \frac{1}{100 \times 100 \times 10^{-6}} = 100\ \Omega$$

So, the net impedance of the circuit may be given as:

$$Z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{(120)^2 + (100 - 10)^2} = 150\ \Omega$$

(capacitive)

The peak value of current flowing through the circuit may be given as:

$$I_0 = \frac{E_0}{Z} = \frac{30}{150} = 0.2\ \text{A}$$

The resonance frequency for the circuit may be given as:

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{100 \times 10^{-3} \times 100 \times 10^{-6}}} = 50.33\ \text{Hz}$$

Problem 7.97: *A 12 Ω resistor and an inductor of (0.05 / π) H with negligible resistance are connected in series. The combination is connected across a 130 V, 50 Hz a.c. source. Determine the current flowing through the circuit and the potential difference across the individual circuit elements.* [CBSE 1999-2000]

Solution: $R = 12\ \Omega$, $L = \frac{0.05}{\pi}$ H, $V = 130$ V, $f = 50$ Hz

The inductive reactance offered by the inductor to the flow of current may be given as:

$$X_L = 2\pi f L = 2\pi \times 50 \times \frac{0.05}{\pi} = 5\ \Omega$$

The impedance of the circuit may be given as:

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{(12)^2 + (5)^2} = 13\ \Omega$$

The current flowing through the circuit may be given as:

$$I = \frac{V}{Z} = \frac{130}{13} = 10\ \text{A}$$

The potential difference across the individual circuit elements may be given as:

$$V_R = I \times R = 10 \times 12 = 120\ \text{V}$$

$$V_L = I \times X_L = 10 \times 5 = 50\ \text{V}$$

Problem 7.98: *A resistor of 5 Ω, an inductor of 50 mH and a capacitor C are connected in series across an a.c. source of 100 V, 50 Hz. The current flowing through the circuit is found to be co-phasor with the supply voltage, determine the capacitance in the circuit and the impedance of the circuit.* [CBSE 1998-99]

Solution: $R = 5\ \Omega$, $L = 50$ mH, $V = 100$ V, $f = 50$ Hz

If the phase angle between the applied a.c. voltage and the circuit current is zero, it indicates that the circuit is resonating at this particular frequency. The capacitance C in the circuit may be given by the expression:

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

or, $$C = \frac{1}{(2\pi f)^2 \times L} = \frac{1}{(2\pi \times 50)^2 \times 50 \times 10^{-3}} = 202.64\ \mu F$$

The impedance of the circuit at resonance ($X_L = X_C$, *i.e.* $X_L - X_C = 0$) may be given as:

$$Z = R = 5\ \Omega$$

Problem 7.99: *A resistor R, an inductor of* $(1 / \pi)$ *H and a capacitor* $(1 / \pi)$ μ*F are connected in series across an a.c. source of constant voltage but variable frequency. Determine the frequency for which the voltage across the resistance is maximum.*

Solution: $L = \frac{1}{\pi}$ H, $C = \frac{1}{\pi}$ μF, V = Constant, f = Variable

We know that the supply voltage appears across the resistor at resonance in an *R-L-C* series circuit, which is the maximum voltage that can appear across the resistance. So, the required frequency is the resonance frequency for this circuit and may be given as:

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{\frac{1}{\pi} \times \frac{1}{\pi} \times 10^{-6}}} = 500\ \text{Hz}$$

Problem 7.100: *An a.c. source of frequency* 50 *Hz is connected to the series combination of a* 50 *mH inductor and a lamp. Determine the required capacitance in series with this combination in order that the lamp glows with maximum brightness.* [CBSE 1999-2000]

Solution: f = 50 Hz, L = 50 mH

The lamp will glow with maximum brightness, only when the voltage across the lamp will become maximum. We know that the supply voltage appears across the resistor at resonance in an *R-L-C* series circuit, which is the maximum voltage that can appear across the lamp. So, the circuit must be resonating at the given frequency. The value of the required capacitance may be given by the expression:

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

or, $$C = \frac{1}{(2\pi f)^2 \times L} = \frac{1}{(2\pi \times 50)^2 \times 50 \times 10^{-3}} = 202.64\ \mu F$$

Problem 7.101: *A* 200 *km long telegraph wire has a capacitance of* 0.014 μ*F/km. If it carries an alternating current of frequency* 50 *kHz, determine the value of inductance required in series with the wire to bring the impedance of the line to its minimum value.* [CBSE 1991-92]

Solution: l = 200 km, c = 0.014 μF/km, f = 50 kHz

The total capacitance of the line may be given as:

$$C = c \times l = 0.014 \times 200 = 2.8\ \mu F$$

The impedance of the line will be minimum ($Z = R$), if the circuit is resonating for the given frequency. So, the value of required inductance in series with the line may be given by the expression:

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

or, $$L = \frac{1}{(2\pi f)^2 \times C} = \frac{1}{(2\pi \times 50 \times 10^3)^2 \times 2.8 \times 10^{-6}} = 3.619\ \mu\text{H}$$

Problem 7.102: *An R-L-C series circuit, having R =* 40 Ω, *L =* 4 *H and C =* 100 μ*F, is connected across an a.c. source of* 220 *V but of variable frequency. Determine: i*) *resonance frequency for this circuit, ii*) *the impedance of circuit and the amplitude of the current flowing through the circuit at resonance, iii*) *r.m.s. potential drop across inductor.* [CBSE 1994-95]

Solution: R = 40 Ω, L = 4 H, C = 100 μF, V = 220 V, f = variable

The resonance frequency for this circuit may be given as:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{4 \times 100 \times 10^{-6}}} = 7.958\ \text{Hz}$$

The impedance of the circuit at resonance may be given as:

$$Z = R = 40\ \Omega$$

The amplitude of the current flowing through the circuit at resonance may be given as:

$$I_0 = \sqrt{2}\ I_{rms} = \sqrt{2} \times \frac{V}{Z} = \sqrt{2} \times \frac{V}{R} = \sqrt{2} \times \frac{220}{40} = 7.778\ \text{A}$$

The r.m.s. value of the potential drop across the inductor may be given as:

$$V_L = I_{rms} \times X_L = \frac{I_0}{\sqrt{2}} \times (2\pi f L) = \frac{7.778}{\sqrt{2}} \times (2\pi \times 7.958 \times 4) = 1100\ \text{V}$$

Problem 7.103: *An R-L-C series circuit, having R =* 25 Ω, *L =* 0.12 *H and C =* 0.48 *mF, is connected across an a.c. source of* 220 *V but of variable frequency. Determine the frequency at which the current flowing through the circuit will become maximum.* [Haryana 2001-02]

Solution: R = 25 Ω, L = 0.12 H, C = 0.48 mF, V = 220 V, f = variable

We know that the current will become maximum through the circuit only at resonance, and the resonance frequency may be given as:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.12 \times 0.48 \times 10^{-3}}} = 20.97\ \text{Hz} \approx 21\ \text{Hz}$$

Problem 7.104: *Determine the capacitive reactance of* 10 μ*F capacitor at a frequency of* 1000 *cycles/sec. Also, determine the inductance required in series with the capacitor to produced resonance in the circuit.* [Haryana 2001-02]

Solution: C = 10 μF, f = 1000 Hz

The capacitive reactance of the capacitor may be given as:

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 1000 \times 10 \times 10^{-6}} = 15.92\ \Omega$$

The required inductance in series with the capacitor to produce the resonance in the circuit may be given by the expression:

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

or, $$L = \frac{1}{(2\pi f)^2 \times C} = \frac{1}{(2\pi \times 1000)^2 \times 10 \times 10^{-6}} = 2.533 \text{ mH}$$

Problem 7.105: *Determine the resonance frequency and the Quality factor of an R-L-C series circuit, having $R = (10 / 3)\ \Omega$, $L = 4\ H$ and $C = 36\ \mu F$. How can the sharpness of the resonance of the circuit be improved by a factor of 2 by reducing its full width at half maximum.*

Solution: $R = 10 / 3\ \Omega$, $L = 4$ H, $C = 36\ \mu$F

The resonance frequency of the circuit may be given as:

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{4 \times 36 \times 10^{-6}}} = 13.267 \text{ Hz}$$

The Quality factor of the circuit may be given as:

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{(10/3)} \times \sqrt{\frac{4}{36 \times 10^{-6}}} = 100$$

Since, the sharpness is to be increased only and without disturbing the resonance frequency. So, we cannot alter the value of L and C. In that case, we may alter the value of R only. So, ***the value of R must be halved to double the sharpness at the resonance***.

7.15 Choke Coil:

Choke Coil: *A choke coil is a simple device, an inductor with a large inductance but a smaller* (*as small as possible*) *resistance, which is generally used to reduce and smoothen the current in an a.c. circuit without any appreciable loss of energy in the circuit*.

Construction: A choke coil is made up of a large number of turns of thick copper conductor (very low resistance due to its thicker cross section) over a laminated iron core (to reduce eddy current losses in the core). The choke coil offers a high inductive reactance ($X_L = 2\,\pi\, f\, L$) to the flow of the a.c. current, so that the current reduces considerably through the circuit. The losses in the choke coil are very small due to its negligible resistance and the laminated iron core.

Working Principle: A choke coil is based on the basic principle that, when an a.c. current flows through a choke coil it offers a high reactance to the flow of current resulting in a smaller current in the circuit and the power dissipated in the choke coil is almost zero as the current flowing in the circuit lags the voltage of the choke coil by almost 90°, *i.e.* $P_{choke\ coil} = V_L\, I \cos 89.999° \approx 0$.

A choke coil (having very small resistance in series with a high inductance, *i.e.* $R \ll L$) connected alone across an a.c. supply source is shown in the Fig. 7.68.

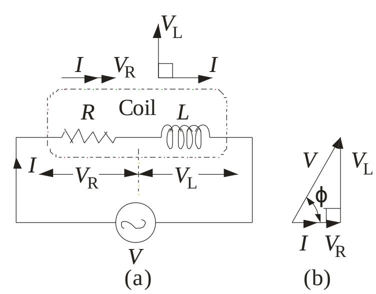


Fig. 7.68

If the rms value of the supply voltage is V and the rms value of current flowing through the choke coil is I, the power dissipated in the circuit (choke coil) may be given as:

$$P_{choke\ coil} = V \times I \times \cos\phi = I\,Z \times I \times \frac{R}{Z} = I^2 R \qquad (7.85)$$

Preference of Choke Coil Over the Ohmic Resistance: Since, ***the resistance of the choke coil is very small* (*as small as possible*)*, so the power dissipated in the choke coil is almost negligible. On the other hand, the impedance of the coil*** $\boldsymbol{Z = \sqrt{R^2 + X_L^2}}$ ***is quite large due to a large inductance of the***

***coil, this helps the choke coil to reduce the circuit current considerably without any appreciable power consumption in the choke coil*.**

The use of choke coil is always preferred over the use of high resistances in the circuit for reducing / controlling the circuit currents, due to the reasons given above.

7.16 Energy Associated with a Pure Inductor: We have already discussed in detail that the power dissipated in an inductor is zero due to the fact that a ***wattles current*** / ***virtual current*** flows through the inductor (lagging the applied voltage by 90°). Since, a virtual current is flowing through the inductor, so it must be drawing / delivering some virtual power from / to the supply source. We have to investigate further into it to understand its exact behavior.

Let us consider the wave form of the applied voltage across a pure inductor and that of the current flowing through the same inductor, lagging the applied voltage by 90°, as shown in the Fig. 7.69. The wave form of voltage (V, dotted line) and of current (I, thicker line) are shown in the Fig 7.69. The current lags behind the voltage by an angle of 90°, for a purely inductive circuit. As, we already know that: $P = V\,I\cos\phi$, the power consumed in the circuit will be zero, whenever either of V or I becomes zero. The points O, $\pi/2$, π, $3\pi/2$ and 2π are such instants / points on the waveform shown in the Fig 7.69, where either V or I is zero. So, the power waveform may be drawn as shown by the thickest line in the Fig. 7.69. We have divided two cycles of power in four zones 1, 2, 3 and 4, each zone lying between two consecutive points where power becomes zero.

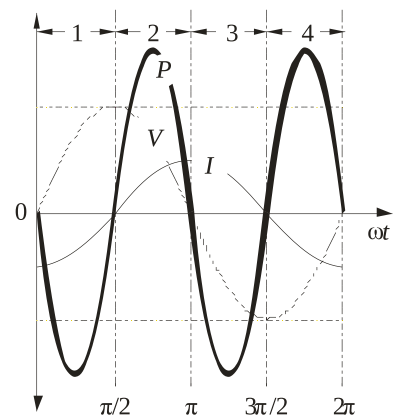


Fig. 7.69

i) Now observe the zone 1, where voltage is positive but current is negative. As the instantaneous power in any a.c. circuit may always be given by $p = v\,i$, so power is negative in zone 1.

ii) Now observe the zone 2, where voltage and current both are positive. So, the power is positive in zone 2.

iii) Now observe the zone 3, where voltage is negative but current is positive. So, the power is again negative in zone 3.

iv) Now observe the zone 4, where voltage and current both are negative. So, the power is again positive in zone 4.

So, the reader may conclude that inductor is behaving as energy storing device, which is taking energy from the supply source in one half of the cycle while returning it back to the circuit during another half of the cycle. In this process the power is pulsating with a frequency double of the frequency of supply source.

We know that the emf induced across an inductor may be given as:

$$E = -N \times \frac{d\phi}{dt} = -L \times \frac{dI}{dt} \qquad (7.86)$$

The work done by the supply source against the induced emf in small time dt may be given as:

$$dW = P\,dt = -E\,I\,dt = L\,I \times \frac{dI}{dt}\,dt = L\,I\,dI \qquad (7.87)$$

So, the energy stored (magnetic energy) in the inductor may be given as:

$$U_B = \int dW = \int_0^{I_0} L\,I\,dI = L \times \left[\frac{I^2}{2}\right]_0^{I_0} = \frac{1}{2}\,L\,I_0^2$$

So, $U_B = \dfrac{1}{2}\, L\, I_0^2$ (7.88)

7.17 Energy Associated with a Pure Capacitor: We have already discussed in detail that the power dissipated in a capacitor is zero due to the fact that a ***wattles current*** / ***virtual current*** flows through the capacitor (leading the applied voltage by 90°). Since, a virtual current is flowing through the capacitor, so it must be drawing / delivering some virtual power from / to the supply source. We have to investigate further into it to understand its exact behavior.

Let us consider the wave forms of the applied voltage across a pure capacitor and the current flowing through the same capacitor, leading the applied voltage by 90°, as shown in the Fig. 7.70. The wave form of voltage (*V*, dotted line) and of current (*I*, thicker line) shown in the Fig 7.70. The current leads the voltage by an angle of 90° for a purely capacitive circuit. As, we already know that: $p = v\, i$, the power consumed in the circuit will be zero, whenever either of *V* or *I* becomes zero. The points *O*, π/2, π, 3π/2 and 2π are such instants / points in the waveform shown in the Fig 7.70, where either *V* or *I* is zero. So, the power waveform may be drawn as shown by the thickest line in the Fig. 7.70. We have divided two cycles of power in four zones 1, 2, 3 and 4, each zone lying between two consecutive points where power becomes zero.

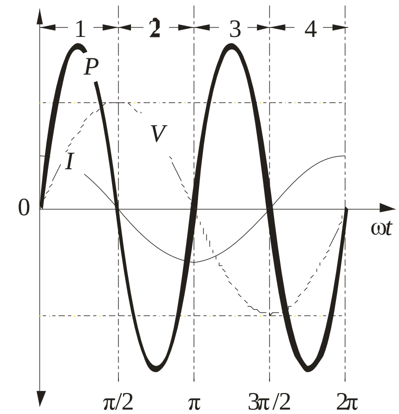


Fig. 7.70

i) Now observe the zone 1, where voltage and current both are positive. As instantaneous power in any a.c. circuit may always be given as $p = v\, i$, so power is positive in zone 1.

ii) Now observe the zone 2, where voltage is positive but the current is negative. So, the power is negative in zone 2.

iii) Now observe the zone 3, where voltage and current both are negative. So, the power is again positive in zone 3.

iv) Now observe the zone 4, where voltage is negative but current is positive. So, the power is again negative in zone 4.

So, the reader may conclude that capacitor is behaving as energy storing device, which is taking energy from the supply source in one half of the cycle while returning it back to the circuit during another half of the cycle. In this process the power is pulsating with a frequency double of the frequency of supply source.

Let us assume that the displacement of charge *q* from one plate to another has already been set up a potential difference *V* between the plates of the capacitor.

So, $V = \dfrac{q}{C}$ (7.89)

The work done by the supply source to transfer a further small charge *dq* from one plate of the capacitor to another plate of the capacitor against the potential difference *V* may be given as:

$$dW = V\, dq = \frac{q}{C}\, dq \qquad (7.90)$$

So, the energy stored (electrostatic energy) in the capacitor may be given as:

$$U_E = \int dW = \int_0^{q_0} \frac{q}{C}\, d\,q = \frac{1}{C} \times \left[\frac{q^2}{2}\right]_0^{q_0} = \frac{1}{2} \times \frac{q_0^2}{C} = \frac{1}{2} \times C\, V^2 = \frac{1}{2} \times q_0\, V$$

So, $U_E = \frac{1}{2} \times \frac{q_0^2}{C} = \frac{1}{2} \times C\,V^2 = \frac{1}{2} \times q_0\,V$ (7.91)

Problem 7.106: *A light bulb (lamp) is rated at 100 W, 220 V a.c. Determine: i) the resistance of the lamp, ii) the peak voltage of the a.c. voltage source, iii) the rms value of current flowing through the lamp.* [NCERT]

Solution: Lamp = 100 W, 220 V

The resistance of the lamp may be given as:

$$R = \frac{V^2}{P} = \frac{(220)^2}{100} = 484\ \Omega$$

The peak value of the a.c. voltage source may be given as:

$$V_0 = \sqrt{2} \times V_{rms} = \sqrt{2} \times 220 = 311.13\ \text{V}$$

The rms value of current flowing through the lamp may be given as:

$$I = \frac{V}{R} = \frac{220}{484} = 0.455\ \text{A}$$

$$= \frac{P}{V} = \frac{100}{220} = 0.455\ \text{A}$$

Problem 7.107: *A resistor and a capacitor are connected in series across an a.c. source. If the potential differences across R and C are 90 V and 120 V respectively and the rms current of the circuit is 3 A, determine the: i) impedance, ii) power factor of the circuit.* [CBSE 2005-06]

Solution: $V_R = 90$ V, $V_C = 120$ V, $I = 3$ A

The value of resistance and capacitive reactance may respectively be given as:

$$R = \frac{V_R}{I} = \frac{90}{3} = 30\ \Omega$$

$$X_C = \frac{V_C}{I} = \frac{120}{3} = 40\ \Omega$$

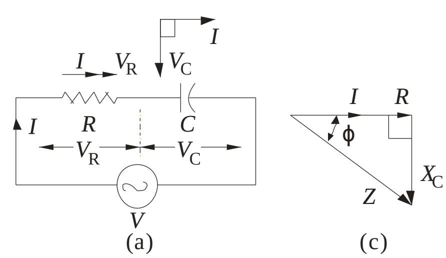


Fig. 7.71

So, the impedance of the circuit may be given as:

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{(30)^2 + (40)^2} = 50\ \Omega$$

The power factor of the circuit, using impedance triangle shown in the Fig. 7.71 (b), may be given as:

$$\text{p.f.} = \cos\phi = \frac{R}{Z} = \frac{30}{50} = 0.6\ \text{(leading)}$$

Problem 7.108: *A resistor of 10 Ω, an inductor of 200 mH and a capacitor C are connected in series across an a.c. source of frequency 50 Hz. If the power factor of the circuit is found to be unity, determine the: i) value of capacitance C, ii) the quality factor of the circuit.* [CBSE 2005-06]

Solution: $R = 10\ \Omega$, $L = 200$ mH, $f = 50$ Hz

The unity power factor of the circuit indicates that the circuit is resonating for the given frequency. So, the value of the capacitance may be given by the expression:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

or, $$C = \frac{1}{(2\pi f)^2 \times L} = \frac{1}{(2\pi\times 50)^2 \times 200\times 10^{-3}} = 50.66\ \mu\text{F}$$

The quality factor of the circuit may be given as:

$$Q = \frac{1}{R} \times \sqrt{\frac{L}{C}} = \frac{1}{10} \times \sqrt{\frac{200\times 10^{-3}}{50.66\times 10^{-6}}} = 6.283$$

$$= \frac{\omega L}{R} = \frac{2\pi f L}{R} = \frac{2\pi\times 50\times 200\times 10^{-3}}{10} = 6.283$$

Problem 7.109: *An alternating voltage source of emf e = 200 sin 300t Volts is applied across a series combination of R = 10 Ω and an inductor of 800 mH. Determine: i) impedance of the circuit, ii) peak value of the current flowing through the circuit, iii) power factor of the circuit.* [CBSE 1933-94]

Solution: e = 200 sin 300t Volts, R = 10 Ω, L = 800 mH

Comparing the expression for emf with the standard emf expression, we get:

E_0 = 200 V, ω = 300 rad/sec

The impedance of the circuit may be given as:

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (\omega L)^2} = \sqrt{(10)^2 + (300\times 800\times 10^{-3})^2} = 240.208\ \Omega$$

The peak value of the current flowing through the circuit may be given as:

$$I_0 = \frac{E_0}{Z} = \frac{200}{240.208} = 0.833\ \text{A}$$

The power factor of the circuit, using the impedance triangle, may be given as:

$$\text{p.f.} = \cos\phi = \frac{R}{Z} = \frac{10}{240.208} = 0.042\ \text{(lagging)}$$

Problem 7.110: *A sin usoidal voltage of peak value 283 V, 50 Hz is applied across an R-L-C series circuit in which R = 3 Ω, L = 25.48 mH and C = 796 μF. Determine: i) the impedance of the circuit, ii) the phase difference between the voltage across the source and the current flowing through the circuit, iii) the power dissipated in the circuit, iv) the power factor of the circuit.* [NCERT]

Solution: V_0 = 283 V, f = 50 Hz, R = 3 Ω, L = 25.48 mH, C = 796 μF

The net reactance of the circuit may be given as:

$$X = (X_L - X_C) = \left(2\pi f L - \frac{1}{2\pi f C}\right)$$

or, $$X = \left(2\pi\times 50\times 25.48\times 10^{-3} - \frac{1}{2\pi\times 50\times 796\times 10^{-6}}\right) = (8 - 4) = 4\ \Omega$$

The impedance of the circuit may be given as:

$$Z = \sqrt{R^2 + X^2} = \sqrt{(3)^2 + (4)^2} = 5\ \Omega$$

The phase difference between the voltage across the source and the current flowing through the circuit, using the impedance triangle shown in the Fig. 7.72 (b), may be given as:

$$\phi = \tan^{-1}\left(\frac{X}{R}\right) = \tan^{-1}\left(\frac{4}{3}\right) = 53.13°$$

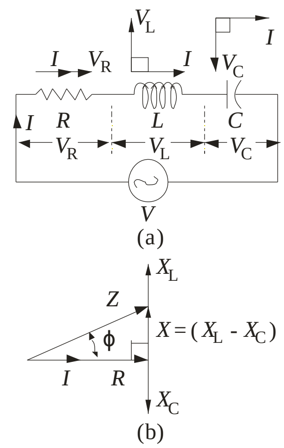


Fig. 7.72

The power dissipated in the circuit may be given as:

$$P = I^2 R = \left(\frac{V_{rms}}{Z}\right)^2 \times R = \left(\frac{V_0}{\sqrt{2}\times Z}\right)^2 \times R = \left(\frac{283}{\sqrt{2}\times 5}\right)^2 \times 3$$

$$= 4805.34 \text{ W} = 4.805 \text{ kW}$$

The power factor of the circuit may be given as:

$$\text{p.f.} = \cos\phi = \frac{R}{Z} = \frac{3}{5} = 0.6 \text{ (lagging)}$$

Problem 7.111: *If the frequency of the source is variable in the previous problem, determine: i) resonance frequency of the given circuit, ii) the impedance, the current flowing through the circuit and the power dissipation in the circuit at resonance.* [CBSE 2001-02]

Solution: $V_0 = 283$ V, $f = 50$ Hz, $R = 3\ \Omega$, $L = 25.48$ mH, $C = 796\ \mu$F

The resonance frequency of the given circuit may be given as:

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{25.48\times10^{-3}\times796\times10^{-6}}} = 35.34 \text{ Hz}$$

The impedance of the circuit at the time of resonance may be given as:

$$Z = R = 3\ \Omega$$

The current flowing through the circuit at the time of resonance may be given as:

$$I = \frac{V_{rms}}{Z} = \frac{V_0}{\sqrt{2}\times R} = \frac{283}{\sqrt{2}\times 3} = 66.704 \text{ A}$$

The power dissipation in the circuit at the time of resonance may be given as:

$$P = I^2 \times R = (66.704)^2 \times 3 = 13.348 \text{ kW}$$

Problem 7.112: *A 200 V variable frequency a.c. source is connected to a series combination of R = 40 Ω, L = 5 H and C = 80 μF. Determine: i) angular frequency of the source to get maximum current in the circuit, ii) the current amplitude at the resonance, iii) the power dissipated in the circuit.* [CBSE 2001-02]

Solution: $V = 200$ V, $f =$ variable, $R = 40\ \Omega$, $L = 5$ H, $C = 80\ \mu$F

The maximum current can be obtained in an *R-L-C* series circuit at its resonance only. The required angular frequency may be given by the expression:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

or, $$\omega = 2\pi f = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5\times80\times10^{-6}}} = 50 \text{ rad/sec}$$

The amplitude of the current at resonance may be given as:

$$I_0 = \frac{V_0}{Z} = \frac{\sqrt{2} \times V_{rms}}{R} = \frac{\sqrt{2} \times 200}{40} = 7.071 \text{ A}$$

The power dissipated in the circuit may be given as:

$$P = I^2 R = \left(\frac{I_0}{\sqrt{2}}\right)^2 \times R = \left(\frac{7.071}{\sqrt{2}}\right)^2 \times 40 = 999.98 \text{ W} \approx 1 \text{ kW}$$

$$= \frac{E_{rms}^2}{R} = \frac{(200)^2}{40} = 1000 \text{ W} = 1 \text{ kW}$$

Problem 7.113: *A current of* 4 *A is flowing through a coil, when it is connected across a supply source of frequency* 50 *Hz. If the power consumed in the coil is* 240 *W and the potential difference across the coil is* 100 *V, determine the inductance of the coil.* [CBSE 1991-92]

Solution: $I = 4$ A, $f = 50$ Hz, $P = 240$ W, $V = 100$ V

The value of the resistance of the coil may be given by the expression:

$$R = \frac{P}{I^2} = \frac{240}{(4)^2} = 15\ \Omega$$

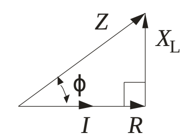


Fig. 7.73

The impedance of the coil may be given by the expression:

$$Z = \frac{V}{I} = \frac{100}{4} = 25\ \Omega$$

So, the inductive reactance of the coil, using impedance triangle shown in the Fig. 7.73, may be given as:

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{(25)^2 - (15)^2} = 20\ \Omega = 2\,\pi\, f\, L$$

or, $$L = \frac{20}{2\pi f} = \frac{20}{2\pi \times 50} = 63.66 \text{ mH}$$

Problem 7.114: *A circuit draws a power of* 550 *W from a source of* 220 *V,* 50 *Hz. The power factor of the circuit is* 0.8 (*lagging*)*. Show that a capacitor of about* $\frac{1}{42\pi} \times 10^{-2}$ *F will have to be connected in series of the circuit to bring the power factor of the circuit to unity.*
[CBSE 1991-92]

Solution: $P = 550$ W, $V = 220$ V, $f = 50$ Hz, p.f. = 0.8 (lagging), $C = \frac{1}{42\pi} \times 10^{-2}$ F

The current flowing through the circuit may be given by the expression:

$$P = V\, I \cos \phi$$

or, $$I = \frac{P}{V \cos \phi} = \frac{550}{220 \times 0.8} = 3.125 \text{ A}$$

The resistance connected in the circuit may be given as:

$$R = \frac{P}{I^2} = \frac{550}{(3.125)^2} = 56.32\ \Omega$$

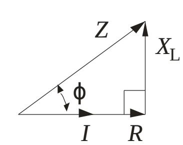


Fig. 7.74

So, the inductive reactance connected in the circuit may be given as:

$$X_L = R \tan\phi = 56.32 \times \frac{\sin\phi}{\cos\phi} = 56.32 \times \frac{\sqrt{1-(0.8)^2}}{0.8} = 42.24\ \Omega$$

The power factor of the circuit will become unity at resonance,

i.e. $X_C = \dfrac{1}{2\pi f C} = X_L = 42.24\ \Omega$

or, $C = \dfrac{1}{2\pi f \times 42.24} = \dfrac{1}{2\pi \times 50 \times 42.24} = \dfrac{1}{42.24\pi} \times 10^{-2}\ \text{F} \approx \dfrac{1}{42\pi} \times 10^{-2}\ \text{F}$

Problem 7.115: *An emf $e = 100 \sin 314t$ Volts is applied across a pure capacitor of 637 μF. Determine: i) the instantaneous current, ii) instantaneous power, iii) the frequency of power, iv) the maximum energy stored in the capacitor.*

Solution: $e = 100 \sin 314t$ Volts, $C = 637$ μF

Comparing the expression for emf with the standard emf equation, we get:

$E_0 = 100$ V, $\omega = 314$ rad/sec

The capacitive reactance of the capacitor at this angular frequency may be given as:

$$X_C = \frac{1}{\omega C} = \frac{1}{314 \times 637 \times 10^{-6}} = 5\ \Omega$$

The instantaneous current may be given as (leading the supply voltage by π/2 rad, as the circuit is purely capacitive):

$$i = \frac{e}{X_C} = \frac{100}{5} \sin(314t + \pi/2) = 20 \sin(314t + \pi/2)\ \text{Amp} = 20 \cos 314t\ \text{Amp}$$

The instantaneous power may be given as:

$$p = v\,i = (100 \sin 314t) \times (20 \cos 314t) = 1000 \times 2 \sin 314t \cos 314t$$
$$= 1000 \sin 628t\ \text{Watt}$$

The frequency of power may be given as:

$$f_p = \frac{\omega_p}{2\pi} = \frac{628}{2\pi} = 100\ \text{Hz (double of the supply frequency)}$$

The maximum energy stored in the capacitor may be given as:

$$U_E = \frac{1}{2} \times C\,E_0^2 = \frac{1}{2} \times 637 \times 10^{-6} \times (100)^2 = 3.185\ \text{J}$$

Problem 7.116: *An R-L-C series circuit is consisting of $R = 100\ \Omega$, $L = \dfrac{2}{\pi}$ H and $C = \dfrac{100}{\pi}$ μF. This series combination is connected across an a.c. source of 220 V, 50 Hz. Determine: i) the impedance of the circuit, ii) the peak value of the current flowing through the circuit, iii) the power factor of the circuit and compare it with the power factor of circuit at resonance.*

Solution: $R = 100\ \Omega$, $L = \dfrac{2}{\pi}$ H, $C = \dfrac{100}{\pi}$ μF, $V = 220$ V, $f = 50$ Hz

The net reactance of the circuit may be given as:

$$X = X_L - X_C = 2\,\pi\,f\,L - \frac{1}{2\pi\,f\,C} = 2\,\pi \times 50 \times \frac{2}{\pi} - \frac{1}{2\pi \times 50 \times (100/\pi) \times 10^{-6}}$$

$$= 200 - 100 = 100\ \Omega$$

The impedance of the circuit may be given as:

$$Z = \sqrt{R^2 + X^2} = \sqrt{(100)^2 + (100)^2} = 100\sqrt{2} = 141.4\ \Omega$$

The peak value of the current flowing through the circuit may be given as:

$$I_0 = \frac{V_0}{Z} = \frac{\sqrt{2}V_{rms}}{Z} = \frac{\sqrt{2} \times 220}{100\sqrt{2}} = 2.2\ \text{A}$$

The power factor of the circuit may be given as:

$$\text{p.f.} = \cos\phi = \frac{R}{Z} = \frac{100}{100\sqrt{2}} = \frac{1}{\sqrt{2}} = 0.707\ \text{(lagging)}$$

The power factor qt resonance is unity for an *R-L-C* series circuit.

So, $\dfrac{\text{p.f.}_{\text{circuit}}}{\text{p.f.}_{\text{resonance}}} = \dfrac{0.707}{1} = 0.707$ or, $\text{p.f.}_{\text{circuit}} : \text{p.f.}_{\text{resonance}} = 0.707 : 1$

Problem 7.117: *The current in a coil of self inductance* 2 *H is increasing according to* $I = 2 \sin t^2$ *Amp. Determine the amount of energy spent during the period when the current changes from zero to* 2 *A.* [Roorkee 1991]

Solution: $L = 2$ H, $I = 2 \sin t^2$ Amp, $I_1 = 0$ A, $I_2 = 2$ A

Since the maximum value of the current is also 2 A.

So, the energy spent may be given as:

$$U_B = \frac{1}{2} \times L\ I_0^2 = \frac{1}{2} \times 2 \times (2)^2 = 4\ \text{J}$$

Problem 7.118: *A* 100 μ*F capacitor is charged with a* 50 *V supply source. Then the source supply is removed and the capacitor is connected across an inductor, as a result of which a current of* 5 *A flows through the inductor. Determine the value of the inductance of inductor.* [CBSE 1991-92]

Solution: $C = 100$ μF, $V = 50$ V, $I = 5$ A

Since inductor and capacitors are the energy storing elements and both exchanges the energy stored in themselves during the half cycles of the oscillations,

So, $$\frac{1}{2} \times C\,V^2 = \frac{1}{2} \times L\,I^2$$

or, $$L = C \times \left(\frac{V}{I}\right)^2 = 100 \times 10^{-6} \times \left(\frac{50}{5}\right)^2 = 0.01\ \text{H} = 10\ \text{mH}$$

Problem 7.119: *A coil has an inductance of* 0.7 *H and is joined in series with a resistor of* 220 Ω. *Determine the watt less component of the current in the circuit, when an alternating emf of* 220 *V at a frequency of* 50 *Hz is applied to this series combination.*

Solution: $L = 0.7$ H, $R = 220\ \Omega$, $V = 220$ V, $f = 50$ Hz

The reactance offered by the coil may be given as:

$$X_L = 2\,\pi\, f\, L = 2\,\pi \times 50 \times 0.7 = 219.91\ \Omega$$

The impedance of the coil may be given as:

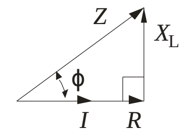


Fig. 7.75

$$Z = \sqrt{R^2 + X^2} = \sqrt{(220)^2 + (219.91)^2} = 311.06\ \Omega$$

The current flowing through the circuit may be given as:

$$I = \frac{V}{Z} = \frac{220}{311.06} = 0.7073\ \text{A}$$

The watt less component of the current through the circuit may be given as:

$$I_{wattless} = I \sin\phi = I \times \frac{X_L}{Z} = 0.7073 \times \frac{219.91}{311.06} = 0.5\ \text{A}$$

Problem 7.120: *An ammeter shows that an alternator is delivering* 20 *A. The voltmeter reads* 220 *V, while a wattmeter shows that* 4 *kW of power is being delivered. Determine the working power factor of the alternator.*

Solution: $I = 20$ A, $V = 220$ V, $P = 4$ kW

The working power factor of the alternator may be given as:

$$\cos\phi = \frac{P}{V\,I} = \frac{4000}{220 \times 20} = 0.909$$

Problem 7.121: *A* 100 Ω *electric iron is connected to* 200 *V,* 50 *Hz a.c. source. Determine average power delivered to the iron, peak power and energy spent in one minute.*

Solution: $R = 100\ \Omega$, $V = 200$ V, $f = 50$ Hz

The average power delivered to the iron may be given as:

$$P = \frac{V^2}{R} = \frac{(200)^2}{100} = 400\ \text{W}$$

The peak power delivered to the iron may be given as:

$$P_0 = \frac{V_0^2}{R} = \frac{(\sqrt{2}\,V_{rms})^2}{R} = \frac{(\sqrt{2} \times 200)^2}{100} = 800\ \text{W}$$

The energy spent in one minute may be given as:

$$U = P \times t = 400 \times 1 \times 60 = 24000\ \text{Watt sec}$$

Problem 7.122: *An a.c. circuit has a resistance of* 100 Ω *and an inductance of* 6 *H connected in series across a* 250 *V,* 50 *Hz a.c. source. Determine the current flowing through the circuit and its power factor.*

Solution: $R = 100\ \Omega$, $L = 6$ H, $V = 250$ V, $f = 50$ Hz

The impedance of the circuit may be given as:

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (2\,\pi\, f\, L)^2} = \sqrt{(100)^2 + (2\,\pi \times 50 \times 6)^2} = 1887.61\ \Omega$$

The current flowing through the circuit may be given as:

$$I = \frac{V}{Z} = \frac{250}{1887.61} = 0.1325\ \text{A} = 132.5\ \text{mA}$$

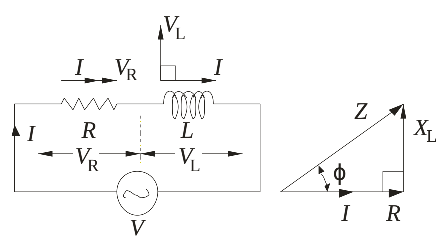


Fig. 7.76

The power factor of the circuit, using the impedance triangle shown in the Fig. 7.76 (b), may be given as:

$$\text{p.f.} = \cos\phi = \frac{R}{Z} = \frac{100}{1887.61} = 0.0529 \text{ (lagging)}$$

Problem 7.123: *An a.c. circuit has a resistance of* 50 Ω, *an inductance of* 10 *H and a capacitor of* 2 μ*F connected in series across a* 220 *V*, 50 *Hz a.c. source. Determine the current flowing through the circuit and its power factor.*

Solution: $R = 50\ \Omega$, $L = 10$ H, $C = 2\ \mu$F, $V = 220$ V, $f = 50$ Hz

The impedance of the circuit may be given as:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(2\pi f L - \frac{1}{2\pi f C}\right)^2}$$

$$= \sqrt{(50)^2 + \left(2\pi \times 50 \times 10 - \frac{1}{2\pi \times 50 \times 2 \times 10^{-6}}\right)^2}$$

$$= 1550.04\ \Omega$$

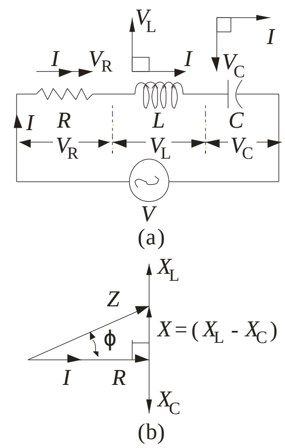


Fig. 7.77

The current flowing through the circuit may be given as:

$$I = \frac{V}{Z} = \frac{220}{1550.04} = 0.1419 \text{ A} = 141.9 \text{ mA}$$

The power factor of the circuit, using the impedance triangle shown in the Fig. 7.77 (b), may be given as:

$$\text{p.f.} = \cos\phi = \frac{R}{Z} = \frac{50}{1550.04} = 0.0323 \text{ (lagging)}$$

Problem 7.124: *A group of electric lamps has a power rating of* 300 *W. An a.c. voltage is applied across the group of lamps, given by* $V = 141.4 \sin(314t + \pi/3)$ *Volts. Determine the effective value of current delivered by the source.*

Solution: $P = 300$ W, $V = 141.4 \sin(314t + \pi/3)$ Volts

The effective value of the current delivered by the source may be given as:

$$I = \frac{P}{V} = \frac{P}{(V_0/\sqrt{2})} = \frac{\sqrt{2} \times 300}{141.4} = 3 \text{ A}$$

Problem 7.125: *An alternating voltage and the corresponding current in an electrical circuit may be given as:*

$e = 110 \sin(\omega t + \pi/6)$ *Volts* *and,* $i = 5 \sin(\omega t - \pi/6)$ *Amp*

Determine the impedance of the circuit and the average power dissipated in the circuit.

Solution: $e = 110 \sin(\omega t + \pi/6)$ Volts, $i = 5 \sin(\omega t - \pi/6)$ Amp

We get following parameters by inspection of the expressions for the emf and the circuit current:

$$E_0 = 110 \text{ V}, \quad I_0 = 5 \text{ A}, \quad \phi = (\pi/6) - (-\pi/6) = \pi/3 = 60°$$

The impedance of the circuit may be given as:

$$Z = \frac{E}{I} = \frac{E_0}{I_0} = \frac{110}{5} = 22\ \Omega$$

The average power dissipated in the circuit may be given as:

$$P = E\,I\cos\phi = \frac{E_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}} \times \cos\phi = \frac{110}{\sqrt{2}} \times \frac{5}{\sqrt{2}} \times \cos 60° = 137.5\ \text{W}$$

Problem 7.126: *An R-L-C series circuit has a potential drop across its individual elements as: V_R = 80 V, V_L = 100 V and V_C = 40 V. Determine the emf of the applied voltage source and the power factor of the circuit.*

Solution: V_R = 80 V, V_L = 100 V, V_C = 40 V

The circuit diagram and the phasor diagram for the voltages across the individual elements connected in series are shown in the Fig. 7.78 (a) and (b) respectively. The emf of the applied voltage source will be equal to the resultant of three voltages connected in series. So, the emf of the source voltage may be given as:

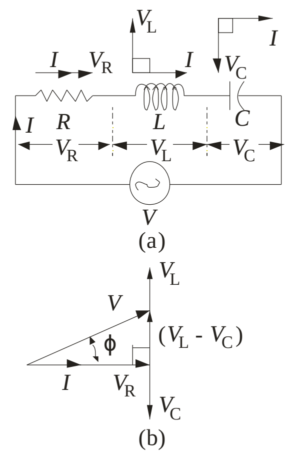


Fig. 7.78

$$V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(80)^2 + (100-40)^2}$$

$$= 100\ \text{V}$$

The power factor of the circuit, using the voltage triangle shown in the Fig. 7.78 (b), may be given as:

$$\text{p.f.} = \cos\phi = \frac{V_R}{V} = \frac{80}{100} = 0.8\ \text{(lagging)}$$

Problem 7.127: *A resistor of* 10 Ω, *an inductor of* 200 *mH and a capacitor of* 500 μ*F are connected in series across a* 100 *V, variable frequency a.c. source. Determine the: i*) *frequency for which the power factor of the circuit will become unity, ii*) *the amplitude of the current at this frequency, iii*) *Quality factor of the circuit at this frequency.*

Solution: R = 10 Ω, L = 200 mH, C = 500 μF, V = 100 V, f = variable

The power factor of the circuit will become unity at resonance only, and the resonance frequency may be given as:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{200\times10^{-3}\times500\times10^{-6}}} = 15.92\ \text{Hz}$$

The amplitude of the current at resonance frequency may be given as:

$$I_0 = \sqrt{2} \times I_{rms} = \sqrt{2} \times \frac{V}{Z} = \sqrt{2} \times \frac{V}{R} = \sqrt{2} \times \frac{100}{10} = 10\sqrt{2}\ \text{A} = 14.14\ \text{A}$$

The quality factor of the circuit at this frequency may be given as:

$$Q = \frac{1}{R} \times \sqrt{\frac{L}{C}} = \frac{1}{10} \times \sqrt{\frac{200\times10^{-3}}{500\times10^{-6}}} = 2$$

$$= \frac{\omega L}{R} = \frac{2\pi f_0 L}{R} = \frac{2\pi\times15.92\times200\times10^{-3}}{10} = 2$$

Problem 7.128: *A resistor of* 10 Ω, *an inductor of unknown value and a capacitor of* 100 μ*F are connected in series across a* 200 *V,* 50 *Hz a.c. source. If the power factor of the circuit is found to be unity,* *determine the value of inductor and the amplitude of the current flowing through the circuit.* [CBSE 2007-08]

Solution: $R = 10\ \Omega$, $C = 100\ \mu F$, $V = 200$ V, $f = 50$ Hz

The unity power factor of the circuit indicates the resonance in the circuit at the given frequency. The value of the inductor may be given by the expression:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

or, $$L = \frac{1}{(2\pi f)^2 \times C} = \frac{1}{(2\pi \times 50)^2 \times 100 \times 10^{-6}} = 101.32 \text{ mH}$$

The amplitude of the current at resonance frequency may be given as:

$$I_0 = \sqrt{2} \times I_{rms} = \sqrt{2} \times \frac{V}{Z} = \sqrt{2} \times \frac{V}{R} = \sqrt{2} \times \frac{200}{10} = 20\sqrt{2} \text{ A} = 28.28 \text{ A}$$

Problem 7.129: *A resistor of* 12 Ω, *an inductor of reactance* 30 Ω *and a capacitor of reactance* 14 Ω *are connected in series across a* 230 *V,* 50 *Hz a.c. source. Determine: i*) *the current flowing through the circuit, ii*) *the phase angle between the applied voltage and the current flowing through the circuit, iii*) *the power factor of the circuit.* [Haryana 2001-02]

Solution: $R = 12\ \Omega$, $X_L = 30\ \Omega$, $X_C = 14\ \Omega$, $V = 230$ V, $f = 50$ Hz

The net reactance of the circuit may be given as:

$$X = X_L - X_C = 30 - 14 = 16\ \Omega \text{ (inductive)}$$

The impedance of the circuit may be given as:

$$Z = \sqrt{R^2 + X^2} = \sqrt{(12)^2 + (16)^2} = 20\ \Omega$$

The current flowing through the circuit may be given as:

$$I = \frac{V}{Z} = \frac{230}{20} = 11.5 \text{ A}$$

The phase angle between the applied voltage and the current flowing through the circuit may be given as:

$$\phi = \tan^{-1}\left(\frac{X}{R}\right) = \tan^{-1}\left(\frac{16}{12}\right) = 53.13°$$

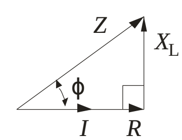


Fig. 7.79

The power factor of the circuit may be given as:

$$\text{p.f.} = \cos\phi = \cos 53.13° = 0.6 \text{ (lagging)}$$

$$= \frac{R}{Z} = \frac{12}{20} = 0.6 \text{ (lagging)}$$

Problem 7.130: *A resistor of* 15 Ω, *an inductor of* 80 *mH and a capacitor of* 60 μ*F are connected in series across a* 230 *V,* 50 *Hz a.c. source. Determine: i*) *the average power transferred to individual circuit element, ii*) *the total power consumed by the circuit.* [Haryana 2000-01]

Solution: $R = 15\ \Omega$, $L = 80$ mH, $C = 60\ \mu F$, $V = 230$ V, $f = 50$ Hz

The current flowing through the circuit may be given as:

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{\sqrt{R^2 + \left(2\pi f L - \frac{1}{2\pi f C}\right)^2}}$$

$$= \frac{230}{\sqrt{(15)^2 + \left(2\pi \times 50 \times 80 \times 10^{-3} - \frac{1}{2\pi \times 50 \times 60 \times 10^{-6}}\right)^2}}$$

$$= \frac{230}{\sqrt{(15)^2 + (25.133 - 53.052)^2}} = \frac{230}{31.693} = 7.257 \text{ A}$$

The power transferred to the individual circuit elements may be given as:

$P_R = I^2 R = (7.257)^2 \times 15 = 789.96$ W

$P_L = 0$ (An inductor does not consume any power at all.)

$P_C = 0$ (A capacitor does not consume any power at all.)

The total power consumed by the circuit may be given as:

$$P = V I \cos\phi = V I \times \frac{R}{Z} = 230 \times 7.257 \times \frac{15}{31.693} = 789.97 \text{ W}$$

$$= I^2 R = (7.257)^2 \times 15 = 789.96 \text{ W}$$

Problem 7.131: *An R-L-C series circuit, having R = 46 Ω, L = 240 mH and C = 240 nF, is connected across a 230 V, variable frequency a.c. source.*

i) *Determine the source frequency for which the amplitude of the current flowing through the circuit is maximum. Also, determine the maximum value.*

ii) *Determine the source frequency for which the average power consumed in the circuit will become maximum. Also, determine the value of maximum power.*

iii) *For which frequencies the power transferred to the circuit will become half of the power consumed at resonance frequency? Determine the amplitude of the current at these frequencies.*

iv) *Determine the Quality factor of the given circuit.*

Solution: $R = 46\ \Omega$, $L = 240$ mH, $C = 240$ nF, $V = 230$ V, f = variable

The current amplitude becomes maximum in the circuit, when the circuit resonates. So, the frequency corresponding to the maximum amplitude of the current in the circuit may be given as:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{240 \times 10^{-3} \times 240 \times 10^{-9}}} = 663.15 \text{ Hz}$$

The amplitude of the maximum value of current flowing through the circuit at this frequency may be given as:

$$I_0 = \sqrt{2} \times I_{rms} = \sqrt{2} \times \frac{V}{Z} = \sqrt{2} \times \frac{V}{R} = \sqrt{2} \times \frac{230}{46} = 5\sqrt{2} \text{ A} = 7.071 \text{ A}$$

The power consumed in the circuit will become maximum, when the current flowing through the circuit will become maximum, *i.e.* at the resonance of the circuit. So, the frequency corresponding to the maximum power consumed by the circuit may be given as:

$$f_{max\ power} = f_0 = 663.15 \text{ Hz}$$

The maximum power consumed by the circuit at this frequency may be given as:

$$P_{max} = V\,I = V \times \frac{I_0}{\sqrt{2}} = 230 \times \frac{5\sqrt{2}}{\sqrt{2}} = 1.15 \text{ kW}$$

$$= I^2 R = \left(\frac{5\sqrt{2}}{\sqrt{2}}\right)^2 \times 46 = 1.15 \text{ kW}$$

The power corresponding to half power frequencies will become half of that consumed by the circuit at resonance frequency. The half power frequencies may be given as:

$$f_{H.P.} = f_0 \pm \frac{R}{4\pi L} = 663.15 \pm \frac{46}{4\pi \times 240 \times 10^{-3}}$$

$$= 663.15 \pm 15.25 = 678.4 \text{ Hz},\ 647.9 \text{ Hz}$$

The current at these frequencies may be given as:

$$P' = \frac{P_{resonance}}{2} = \frac{I_0^2 R}{2} = \left(\frac{I_{resonance}}{\sqrt{2}}\right)^2 \times R = (I')^2 \times R$$

So, $$I'_0 = \frac{I_{resonance}}{\sqrt{2}} = \frac{I_0}{\sqrt{2}} = \frac{5\sqrt{2}}{\sqrt{2}} = 5 \text{ A}$$

The quality factor of the circuit at this frequency may be given as:

$$Q = \frac{1}{R} \times \sqrt{\frac{L}{C}} = \frac{1}{46} \times \sqrt{\frac{240 \times 10^{-3}}{240 \times 10^{-9}}} = 21.739$$

$$= \frac{\omega L}{R} = \frac{2\pi f_0 L}{R} = \frac{2\pi \times 663.15 \times 240 \times 10^{-3}}{46} = 21.739$$

7.18 ***L-C* Oscillations:** *If a charged capacitor is allowed to discharge through a non-resistive inductor* (*pure inductor*), *electrical oscillations of constant amplitude and frequency are produced.* These oscillations are known as ***L-C Oscillations***.

Analysis of Production of *L-C* Oscillations:

Qualitative Analysis of *L-C* Oscillations: Consider the Fig.7.80 (a), in which a fully charged capacitor (*C*) is shown connected across a pure inductor (*L*) through a switch (*S*). When the switch (*S*) is open the current in the circuit is zero. The electrical energy stored in the capacitor at this instant may be given as:

$$U_E = \frac{1}{2} \times \frac{q_0^2}{C} = \frac{1}{2} \times C\,V_0^2 = \frac{1}{2} \times q_0\,V \qquad (7.92)$$

As soon as the switch (S) is closed, as shown in the Fig. 7.80 (b), a current starts to build up in the circuit due to potential difference (Electric field) across the capacitor, the current goes on increasing with the time (t). This increasing current starts to build up an induced emf across the inductor, hence the electrical energy stored in the capacitor (U_E) is being transferred to the inductor with the help of the current flowing through the circuit and now the energy is being stored in the form of magnetic energy in the inductor (U_B). The magnetic energy stored in the inductor at any instant may be given as:

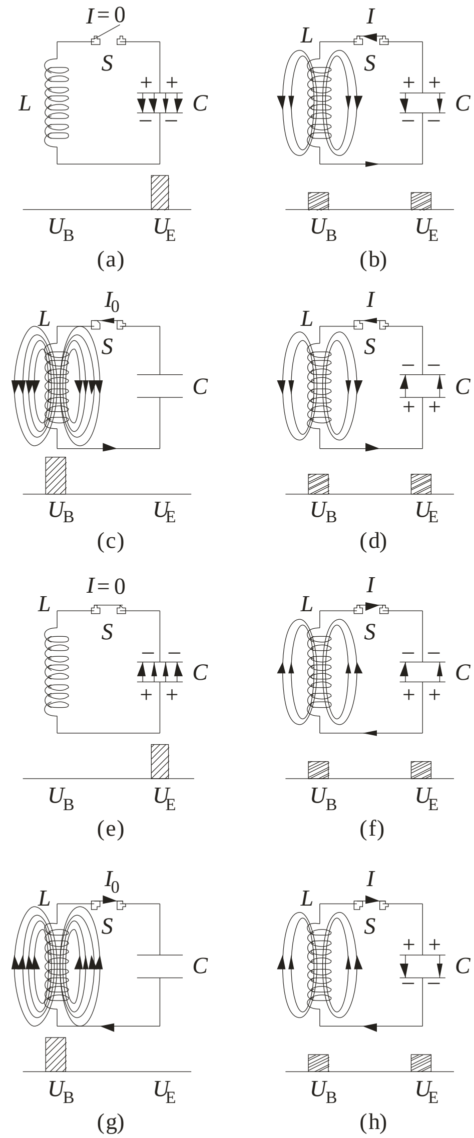


Fig. 7.80

$$U_B = \frac{1}{2} \times L\, I^2 \tag{7.93}$$

The current increases up to some maximum value (I_0) corresponding to the entire electrical energy transferred from the capacitor (U_E) to the inductor in the form of magnetic energy (U_B), as shown in the Fig. 7.80 (c). The capacitor has been fully discharged now, but the current in the circuit is at its maximum value (I_0) and the inductor has the maximum induced emf across its terminals at this instant. The magnetic energy stored in the inductor at this instant may be given as:

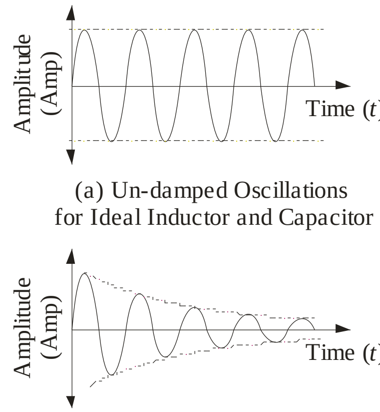
$$U_B = \frac{1}{2} \times L\; I_0^2 \tag{7.94}$$

The magnetic flux linked with the inductor starts to decrease after this instant, inducing a current (but of decreasing nature) in the same direction as the earlier current, as shown in the Fig. 7.80 (d). So, this decreasing current in the same direction starts to charge the capacitor but in opposite direction to its earlier charging state [the reader may note that the sign on the capacitor plates has been reversed now in Fig. 7.80 (d)]. The magnetic energy stored in the inductor is now being converted into the electrical energy (U_E) to be stored in the capacitor. This process continues with decreasing current till the capacitor gets fully charged in opposite direction to that of its earlier charged state, as shown in the Fig. 7.80 (e). The current in the circuit becomes zero at this state corresponding to the entire magnetic energy (U_B) transferred from the inductor to the capacitor in the form of electric energy (U_E), as shown in the Fig. 7.80 (e). The capacitor has been fully charged now, and the current in the circuit is at zero value (I = 0), so the capacitor has the maximum potential difference across its terminals at this instant. The electrical energy (U_E) stored in the capacitor at this instant may be given as:

$$U_E = \frac{1}{2} \times \frac{q_0^2}{C} = \frac{1}{2} \times C\, V_0^2 = \frac{1}{2} \times q_0\, V \tag{7.95}$$

The capacitor starts to discharge through the inductor again by sending a current in opposite direction to that of the earlier, as shown in the Fig. 7.80 (f). The electrical energy (U_E) is once again being transferred from the capacitor to the inductor to be stored in the form of magnetic energy (U_B). And so, the process repeats in the opposite direction, as shown in the Fig. 7.80 (e), (f), (g), (h) and finally (a). So, repeating the process in opposite direction the system (L-C) returns to its initial state, as shown in the Fig. 7.80 (a).

The energy of system continuously surges between the capacitor and the inductor back and forth from the electric field of the capacitor to the magnetic field of the inductor. This will result in the production of the electrical oscillations of a constant frequency f_0, and are known as *L-C oscillations*. If the inductor and capacitor are pure in nature (*i.e.* no resistance is associated with them, which is not possible in practice), there will be no loss of energy in the circuit and hence the amplitude of the oscillations remains constant over the time, as shown in the Fig. 7.81 (a). Such oscillations are known as ***Un-damped Oscillations***.



(a) Un-damped Oscillations for Ideal Inductor and Capacitor

(b) Damped Oscillations for Real Inductor and Capacitor

Fig. 7.81

However, the *L-C* oscillations are ***damped oscillations*** in nature due to the following two reasons:

i) A real inductor as well as a real capacitor and also the connecting leads always have a small resistance associated with them, so the energy stored in the system will decay slowly (due to power losses in resistance of inductor and capacitor) resulting in decreasing amplitude of the oscillations, and finally oscillations will diminish with the elapsing time. Such oscillations are known as ***Damped Oscillations***, as shown in the Fig. 7.81 (b).

ii) The total energy of the system would not remain constant even in the absence of the resistance of inductor, capacitor and connecting leads. The energy will radiate away in the form of electromagnetic radiations / waves. The working of radio and television transmitters is based on such radiations.

Quantitative Analysis of *L-C* Oscillations: Consider a capacitor (C), initially charged to q_0, and connected to an inductor (L) through a switch (S), as shown in the Fig. 7.82. When the switch (S) is closed at time $t = 0$, the charge on the capacitor begins to flow through the circuit from positive plate of the capacitor to the negative plate of the capacitor, which gives rise to a gradually increasing current in the circuit. This gradually increasing current will decrease the charge on the capacitor and induces an emf across the inductor.

The potential difference across the capacitor at any instant of time (t) may be given as:

$$V = \frac{q}{C} \tag{7.96}$$

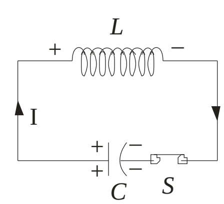


Fig. 7.82

The current in the circuit at any instant of time (t) may be given as:

$$I = -\frac{dq}{dt} \tag{7.97}$$

(The negative sign is due to the decreasing charge with increasing current)

The induced emf across the inductor (L) at any instant of time (t) may be given as:

$$e = -L\,\frac{dI}{dt} \tag{7.98}$$

Applying *Kirchhoff's Voltage Law* to the *L-C* circuit:

$$-L\,\frac{dI}{dt} + \frac{q}{C} = 0$$

or, $$-L\,\frac{d}{dt}\left(-\frac{dq}{dt}\right) + \frac{q}{C} = 0$$

or, $$\frac{d^2 q}{dt^2} + \frac{1}{LC} \times q = 0$$

or, $$\frac{d^2 q}{dt^2} + \omega_0^2 \, q = 0 \qquad \left(\text{Where, } \omega_0 = \frac{1}{\sqrt{LC}}\right) \tag{7.99}$$

Above equation is linear differential equation of second order, which will have a general solution of the form given by:

$$q = A \cos \omega_0 t + B \sin \omega_0 t \tag{7.100}$$

At time $t = 0$, $q = q_0$,

So, $q_0 = A \cos 0 + B \sin 0$

or, $$A = q_0 \tag{7.101}$$

Differentiating equation (7.100) *w.r.t.* the time:

$$\frac{dq}{dt} = -A \sin \omega_0 t + B \cos \omega_0 t \tag{7.102}$$

At time $t = 0$, $q = q_0$ (maximum), So, $I = \frac{dq}{dt} = 0$.

So, $0 = -A \sin 0 + B \cos 0$

or, $$B = 0 \tag{7.103}$$

Putting equations (7.101) and (7.103) in equation (7.100):

$$q = q_0 \cos \omega_0 t \tag{7.104}$$

and, $$I = -\frac{dq}{dt} = \omega_0 \, q_0 \sin \omega_0 t = I_0 \sin \omega_0 t \tag{7.105.1}$$

Where, $$I_0 = \omega_0 \, q_0 \tag{7.105.2}$$

So, the charge on the capacitor plates and hence the current in the *L*-*C* oscillator varies simple harmonically, between its positive maximum value to negative maximum value, with time and having an angular frequency given by:

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{7.106}$$

And hence, the frequency of oscillations may be given as:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \tag{7.107}$$

Conservation of Energy in *L*-*C* Oscillations: The total energy in an *L*-*C* oscillator (*the sum of energies stored in the inductor and that in the capacitor*) remains constant at any instant of time, as similar to that of a *Simple Harmonic Motion*.

The total energy in an *L*-*C* oscillator at any instant of time may be given as:

$$U = U_E + U_B = \frac{1}{2} \times \frac{q^2}{C} + \frac{1}{2} \times L I^2 \tag{7.108}$$

We know that: $q = q_0 \cos \omega_0 t$, and, $I = -\dfrac{dq}{dt} = \omega_0\, q_0 \sin \omega_0 t$

So, $$U = \frac{1}{2} \times \frac{(q_0 \cos \omega_0 t)^2}{C} + \frac{1}{2} \times L\,(\omega_0\, q_0 \sin \omega_0 t)^2 = \frac{q_0^2}{2C} \times \cos^2 \omega_0 t + \frac{1}{2} \times L\ \omega_0^2\ q_0^2\ \sin^2 \omega_0 t$$

$$= \frac{q_0^2}{2C} \times \cos^2 \omega_0 t + \frac{1}{2} \times L\ \frac{1}{LC}\ q_0^2\ \sin^2 \omega_0 t = \frac{q_0^2}{2C} \times \cos^2 \omega_0 t + \frac{q_0^2}{2C} \times \sin^2 \omega_0 t$$

$$= \frac{q_0^2}{2C} \times (\cos^2 \omega_0 t + \sin^2 \omega_0 t) = \frac{1}{2} \times \frac{q_0^2}{C} = \frac{1}{2} \times C\ V_0^2 = \text{Initial energy} \quad (7.109)$$

So, *the total energy stored in any L-C oscillator remains constant irrespective of the time*.

Mechanical Analogy for *L-C* Oscillations: The *L-C* oscillations are quite similar to the mechanical oscillations of a mass-spring system.

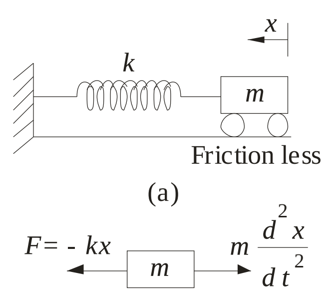
Consider the mass spring system shown in the Fig. 7.83 (a) on a friction less plane. The oscillating body has a mass *m* and the spring has a spring constant of *k*. If the displacement of the oscillating mass (*m*) is *x* at any instant of the time *t*, the force experienced by the mass due to the spring towards its mean position may be given as:

$$F = -k\,x \qquad (7.110)$$

The (−)ve sign shows that the restoring force always tends towards the mean position of the oscillating mass.

The acceleration of the mass (*m*) at this instant of the time / the opposing force due to the moment of inertia of the mass (*m*) may be given as:

$$F = m\,a = m\,\frac{d^2 x}{dt^2} \qquad (7.111)$$



(a)
(b)
Fig. 7.83

The free body diagram (According to *D'Alembert's Principle*) of the oscillating mass (*m*) is drawn in the Fig. 7.83 (b). The reader may observe that two forces will be equal and opposite to each other at equilibrium.

So, $$F = -k\,x = m\,\frac{d^2 x}{dt^2}$$

or, $$m\,\frac{d^2 x}{dt^2} + k\,x = 0$$ or, $$\frac{d^2 x}{dt^2} + \frac{k}{m}\,x = 0$$

or, $$\frac{d^2 x}{dt^2} + \omega_0^2\ x = 0 \qquad \text{Where,} \quad \omega_0 = \sqrt{\frac{k}{m}} \qquad (7.112)$$

The equations for the *L-C oscillations* were found to be:

$$\frac{d^2 q}{dt^2} + \omega_0^2\ q = 0 \qquad \left(\text{Where, } \omega_0 = \frac{1}{\sqrt{LC}}\right) \qquad (7.113)$$

On comparison of above equations (7.112) and (7.113), the reader may find following analogies in two different systems:

i) The charge (q) in the electrical system is analogous to the displacement (x) in the mechanical system.

ii) The current $\left(I = \frac{dq}{dt}\right)$ in the electrical system is analogous to the velocity $\left(v = \frac{dx}{dt}\right)$ in mechanical system.

iii) The inductance (L) in the electrical system is analogous to the mass (m) in the mechanical system.

iv) The reciprocal of capacitance $\left(\frac{1}{C}\right)$ in the electrical system is analogous to the spring constant (k) in the mechanical system.

Above analogy may be remembered in a tabular form as given below:

SR. NO.	ELECTRICAL SYSTEM	MECHANICAL SYSTEM
1	Charge (q)	Displacement (x)
2	Current $\left(I = \frac{dq}{dt}\right)$	Velocity $\left(v = \frac{dx}{dt}\right)$
3	Inductance (L)	Mass (m)
4	Reciprocal of Capacitance $\left(\frac{1}{C}\right)$	Spring Constant (k)
5	$\omega_0 = \frac{1}{\sqrt{LC}}$	$\omega_0 = \sqrt{\frac{k}{m}}$
6	Electrical differential equation: $\frac{d^2q}{dt^2} + \omega_0^2\, q = 0$	Mechanical differential equation: $\frac{d^2x}{dt^2} + \omega_0^2\, x = 0$

Problem 7.132: *Determine the wavelength of radio waves radiated from a circuit containing an inductor of* 8 μ*H and a capacitor of* 0.02 μ*F in series.* [Haryana 1191-92]

Solution: L = 8 μH, C = 0.02 μF

The frequency of oscillations of the circuit may be given as:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{8\times10^{-6}\times0.02\times10^{-6}}} = 397.887 \text{ kHz}$$

The wavelength of the radio waves radiated from this *L-C Oscillator* may be given as:

$$\lambda = \frac{C}{f_0} = \frac{3\times10^8}{397.887\times10^3} = 753.983 \text{ m}$$

Problem 7.133: *An inductor of* 2 *mH is connected across a charged capacitor of* 5 μ*F and the resulting L-C circuit is set oscillating at its natural frequency of oscillations. Let q denotes the instantaneous charge on the capacitor and I be the current in the circuit. It is found that the maximum value of charge q is* 200 μ*C. Determine:*

i) *The value of* $\frac{dI}{dt}$, *when q* = 100 μ*C.*

ii) *The value of I, when q* = 200 μ*C.*

iii) *The maximum value of the current flowing through the circuit.*

iv) *The value of q, at the instant when the current in the circuit is at half of its maximum value.* [IIT 1998]

Solution: L = 2 mH, C = 5 μF, q_0 = 200 μC

The natural angular frequency of oscillations may be given as:

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2\times10^{-3}\times5\times10^{-6}}} = 1\times10^4 \text{ rad/sec}$$

We know that the charge on the capacitor, the circuit current and the rate of change of current in an *L-C Oscillator* may respectively be given as:

$$q = q_0 \cos \omega t$$

and, $$I = -\frac{dq}{dt} = \omega_0\, q_0 \sin \omega t = I_0 \sin \omega t$$

and, $$\frac{dI}{dt} = \omega_0^2\, q_0 \cos \omega t = \omega_0^2\, q$$

When, q = 100 μC, the value of $\frac{dI}{dt}$ may be given as:

$$\frac{dI}{dt} = \omega_0^2\, q = (1\times10^4)^2 \times 100\times10^{-6} = 1\times10^4 \text{ A/sec}$$

When, q = 200 μC, the corresponding time is t = 0 sec, *i.e.* start of the discharging process:

So, $I = \omega_0\, q_0 \sin \omega t = \omega_0\, q_0 \sin 0 = 0$ Amp

The maximum value of the current flowing through the circuit may be given as:

$$I_0 = \omega_0\, q_0 = 1\times10^4\times200\times10^{-6} = 2 \text{ A}$$

When the current in the circuit is at half of its maximum value, the current expression may be given as:

$$I = I_0 \sin \omega t = \frac{I_0}{2}$$

So, $$\sin \omega t = \frac{1}{2} = 0.5$$

Now, $$q = q_0 \cos \omega t = q_0 \times \sqrt{1-\sin^2 \omega t} = 200 \times \sqrt{1-(0.5)^2} = 173.205 \text{ μC}$$

Problem 7.134: *A coil of inductance* 150 *mH is connected in series with a variable capacitor of capacitance* 20 *pF to* 500 *pF. Determine the frequency range over which the circuit may be tuned.*

Solution: L = 150 mH, C = 20 pF –to– 500 Pf

The two extreme frequencies may respectively be given as:

$$f_1 = \frac{1}{2\pi\sqrt{LC_1}} = \frac{1}{2\pi\sqrt{150\times10^{-3}\times20\times10^{-12}}} = 91.888 \text{ kHz}$$

and, $f_2 = \dfrac{1}{2\pi\sqrt{LC_2}} = \dfrac{1}{2\pi\sqrt{150\times10^{-3}\times500\times10^{-12}}}$ = 18.378 kHz

Problem 7.135: *A* 10 μ*F capacitor is charged to a potential of* 25 *V. The battery is then disconnected and a purely inductive coil of* 100 *mH is connected across the capacitor, so that L-C oscillations may setup in the circuit. Determine the maximum current in the coil.*

Solution: C = 10 μF, V = 25 V, L = 100 mH

We know that the charge on the capacitor and the circuit current in an *L-C Oscillator* may respectively be given as:

$$q = q_0 \cos \omega t \qquad \text{and,} \qquad I = -\frac{dq}{dt} = \omega_0\, q_0 \sin \omega t = I_0 \sin \omega t$$

So, the maximum current in the circuit may be given as:

$$I_0 = \omega_0\, q_0 = \frac{1}{\sqrt{LC}} \times C\,V = V \times \sqrt{\frac{C}{L}} = 25 \times \sqrt{\frac{10\times10^{-6}}{100\times10^{-3}}} = 0.25 \text{ A}$$

Problem 7.136: *A* 1.5 *mH inductor in an L-C circuit stores a maximum energy of* 30 μ*J. Determine the maximum current in the circuit.*

Solution: L = 1.5 mH, $U_{\max}$ = 30 μJ

We know that the energy stored in an *L-C* oscillator remains constant at all the times and may be given as:

$$U_{max} = \frac{1}{2} \times L\, I_0^2 = \frac{1}{2} \times \frac{q_0^2}{C}$$

So, $I_0 = \sqrt{\dfrac{2U_{\max}}{L}} = \sqrt{\dfrac{2\times30\times10^{-6}}{1.5\times10^{-3}}}$ = 0.2 A

Problem 7.137: *In an L-C oscillator circuit, the self inductance of the coil used is* 10 *mH. If the natural frequency of oscillation of the circuit is* 1 *MHz, determine the capacitance of the required capacitor in the circuit.*

Solution: L = 10 mH, f_0 = 1 MHz

The natural frequency of oscillations in an *L-C* oscillator circuit may be given as:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

or, $C = \dfrac{1}{(2\pi f)^2 \times L} = \dfrac{1}{(2\pi\times1\times10^6)^2\times10\times10^{-3}}$ = 2.533 pF

Problem 7.138: *An electromagnetic wave of wavelength* 300 *m can be produced by a transmitter. A capacitor of capacitance* 2.5 μ*F is available. Determine the required inductance of the coil for the L-C oscillatory circuit.*

Solution: λ = 300 m, C = 2.5 μF

The frequency of the generated wave may be given by the expression:

$$f_0 = \frac{c}{\lambda} = \frac{3\times10^8}{300} = 1 \times 10^6 \text{ Hz} = 1 \text{ MHz}$$

The natural frequency of oscillations in an *L*-*C* oscillator circuit may be given as:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

or, $$L = \frac{1}{(2\pi f)^2 \times C} = \frac{1}{(2\pi \times 1 \times 10^6)^2 \times 2.5 \times 10^{-6}} = 10.132 \text{ nH}$$

7.19 Transformer: *A transformer is a static electrical machine. It is being used for two main purposes: i*) *To Change the voltage and current level of electricity* (*electric power supply*) *for transmission and distribution purposes, ii*) *Electrical isolation of two electrical circuits for safety from high voltage surges of either side*.

If, it is used for increasing the voltage level it is known as ***step-up transformer***, on the other hand if it is used for decreasing the voltage level it is known as ***step-down transformer***.

Principle of Operation: *A transformer works on the principle of mutual induction, i.e. if a changing current is flowing through one of the two inductively coupled windings, an emf is being induced in the secondary winding* (*as well as in the primary winding*) *according to the* ***Faraday's Law of Electromagnetic Induction***. We already have discussed it in detail, in article 6.15 in the chapter *Electromagnetic Induction*.

Construction: Every transformer has primary and secondary windings to change the voltage and current levels at two sides of the transformer, however frequency of the supply remains same on both the sides, on the primary as well as on the secondary side.

The soft iron core, on which two windings (primary winding and secondary winding) of the transformer are wound, is made up of *laminated punching / stamping in order to reduce the eddy current losses* in the transformer. The soft iron core has a *very high relative permeability*, due to which the entire flux produced (of considerable amount) due to the primary winding is strictly confined within the core of the transformer only and hence links both of the windings of the transfer, primary as well as secondary windings. This prevents the stray currents being generated in the conductors lying in the vicinity of a transformer and the consequent power losses.

The transformer core and the windings are immersed in the transformer oil of *high dielectric strength* inside the tank of the transformer. This transformer oil serves two purposes for the transformer:

i) It provides high insulation resistance (due to its high dielectric strength), between the turns of windings itself as well as turns of windings to the iron core and iron body of transformer.

ii) It provides cooling to the transformer windings by extracting heat from the transformer windings.

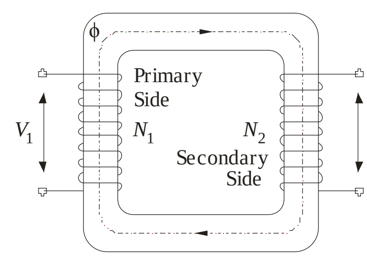


Fig. 7.84 Core type Single Phase Transformer

Transformers have two types of construction: *a*) ***Core type transformer***, *b*) ***Shell type transformer***.

***a*) Core Type Transformer:** *The iron core of the transformer is surrounded by the windings of the transformer* in this type of the construction. The windings are wounded on the two limbs of the core, as shown in the Fig. 7.84.

***b*) Shell Type Transformer:** *Primary and secondary windings of the transformer are surrounded by the core of the transformer* in this type of the construction. The windings are wounded on the central limb of the core, as shown in the Fig. 7.85.

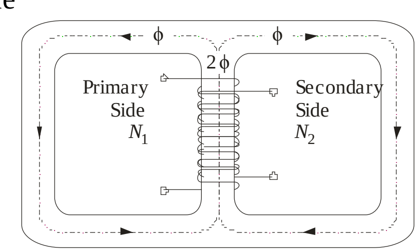


Fig. 7.85 Shell type Single Phase Transformer

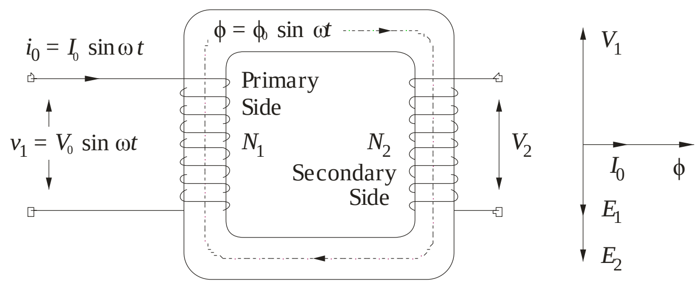
7.20 Working of A Transformer: A transformer has two windings, wound on the same iron core, *i.e.* two magnetically coupled windings. When primary winding of the transformer is excited by an alternating voltage, an alternating magnetizing current lagging the applied voltage by almost 90° starts to flow in the primary winding (as the primary windings and the secondary windings are highly inductive). An alternating flux, co-phasor with the magnetizing current, is set-up in the transformer core. This alternating flux is associated with both the windings of the transformer, primary as well as secondary windings. An emf is being induced in both the windings E_1 (*back emf*) in primary winding and E_2 in secondary winding (depending on the number of turns in each winding) due to this alternating flux. The electrical energy may now be transferred from primary winding to secondary winding with the help of magnetic coupling between primary and secondary windings with-out having any electrical contact / connection between them.

Transformer on D.C.: When primary winding of a transformer is being excited with the help of a *D.C.* voltage source, a uni-directional flux is set-up in the core and hence no "*back emf*" is induced in the primary winding and no emf is induced in the secondary winding, so we get zero output at the terminals of secondary winding. Also as, there is no induced "*back emf*" in the primary winding and the winding resistances are quite low, consequently a high current will flow in the primary winding of the transformer and the primary winding will get burnt off. The transformer will get permanently damaged and become un-usable.

7.21 EMF Equation of a Transformer: Let us assume the primary winding of the transformer as purely inductive and resistance of the winding is zero. Now, if we excite the primary winding by a voltage given as:

$$v_1 = V_0 \sin \omega t \qquad (7.114)$$

A current (*magnetization current*) will set-up in the primary winding, which is almost 90° lagging behind the supply voltage v_1, and may be given as:



(a) (b)

Fig. 7.86 EMF Induced

$$i_1 = I_0 \sin (\omega t - 90°) = I_0 \cos \omega t \qquad (7.115)$$

The flux set-up in the transformer core by this magnetizing current is co-phasor with the current and may be given as:

$$\phi_1 = \phi_0 \cos \omega\, t \qquad (7.116)$$

Now, the emf induced in any winding (primary / secondary winding) due to this alternating flux may be given as:

$$e = -N \frac{d\phi}{dt} = -N \frac{d}{dt} (\phi_0 \cos \omega\, t) = N\,\omega\,\phi_0 \sin \omega\, t \qquad \text{(again, 90° lagging behind the flux)}$$

$$e = N\,\omega\,\phi_0 \sin \omega\, t = e_0 \sin \omega\, t \qquad (7.117)$$

Hence, $$E = \frac{E_0}{\sqrt{2}} = \frac{N\,\omega\,\phi_0}{\sqrt{2}} = \frac{2\pi f N \phi_0}{\sqrt{2}} = 4.44 \times f N \phi_0 \qquad (7.118)$$

So, $$E_1 = 4.44 \times f N_1 \phi_0 \qquad (7.119)$$

and, $$E_2 = 4.44 \times f N_2 \phi_0 \qquad (7.120)$$

Since, the transformer is assumed as an ideal transformer (having zero winding resistances and leakage reactance), So, $V_1 = E_1$ and, $V_2 = E_2$

Now, $$\frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{V_2}{V_1} \quad (7.121)$$

We know that, for an ideal transformer (having no losses due to its zero winding resistances), power input to the transformer must be equal to the power output of the transformer.

So, $V_1 I_1 = V_2 I_2$ or, $$\frac{V_2}{V_1} = \frac{I_1}{I_2} \quad (7.122)$$

So now equation (7.121) becomes,

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{I_1}{I_2} \quad (7.123)$$

Where, $\frac{E_2}{E_1} \rightarrow$ Ratio of two emf's

$\frac{N_2}{N_1} \rightarrow$ Transformation ratio / Turns ratio

$\frac{V_2}{V_1} \rightarrow$ Ratio of Output Voltage to Input Voltage

$\frac{I_1}{I_2} \rightarrow$ Ratio of Input Current to Output Current

The equations (7.122) and (7.123) revel that a step-up transformer steps up the voltage, while at the same time steps down the current exactly in the same ratio, so that the power on the two sides of a transformer remains balanced. On the other hand, a step down transformer steps down the voltage, while at the same time steps up the current exactly in the same ratio, so that the power on the two sides of a transformer remains balanced.

The efficiency of a real transformer is fairly large in the range of 90% – 99%, but not 100%.

Above derivation, for efficiency of the transformer, may be done by using following assumptions (deviation from the real transformer):

i) The primary as well as secondary windings have negligible resistance (R_1 and $R_2 \approx 0$).

ii) The total flux created by the primary winding is linked also with the secondary winding.

iii) The secondary winding is open circuited, *i.e.* not supplying any load current to the load.

7.22 **Efficiency of a Transformer:** The efficiency of a transformer may be given as:

$$\eta = \frac{\text{Power Output}}{\text{Power Input}} \times 100\% \quad (7.124)$$

Losses in the Transformers: There are various types of losses in a transformer, which results in the heating (temperature rise) of the transformer during its duty cycle.

***i*) Copper Losses (*Variable Losses*):** They occur in the copper windings, primary as well as secondary windings (*made up of copper wires*), of the transformer as $I^2 R$ losses (*heating effect of the current*). They are variable losses, as current of the transformer may vary from *No-load* to *Full-load* and $I^2 R$ losses will also change with current as [$P_{Cu} \propto I^2$].

ii*) Iron / Core Losses (*Constant Losses*):** They occur in the core (*made up of iron*) of the transformer. They are constant losses, independent of the load on transformer, ***as flux in the core of transformer remains constant for all values of loads. Iron losses are of two types:

***a*) Hysteresis Losses:** The work done lost in aligning the small atomic dipoles of iron in either direction is known as *hysteresis loss*. The area of hysteresis loop (*B-H* Loop) represents the *hysteresis loss* in the iron core occurred in one cycle of the current, as shown in the Fig. 7.87 (a) $[\{P_h \propto f\,(\phi_m)^x\}$, where x is between 1.5 – 2.5].

***b*) Eddy Current Losses:** The circular currents, induced in the iron body internally due to the induced emf in iron because of the alternating flux in the core, are known as "*eddy currents*". The eddy currents cause heat losses in the iron body. The eddy currents also oppose the cause due to which they are being induced. The induction and flow of eddy currents internally in the iron body is shown in the Fig. 7.87 (b) $[P_e \propto f^2\,(\phi_m)^2]$.

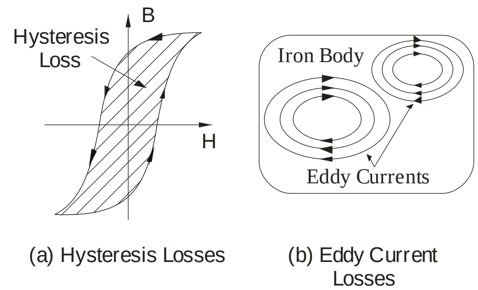


Fig. 7.87

iii*) Humming Losses:** The dimensions of the transformer core changes slightly along their length, width and thickness due to the alternating nature of the flux. *The phenomenon of change in dimensions of a magnetic material subjected to a magnetic field is known as* ***"Magnetostriction". The transformer creates the humming noise during its operation due to the *magnetostriction*, which is the loss of energy in the form of sound.

iv*) Flux Leakage:** Some of the flux created by the primary winding leaks through the air in the vicinity of the primary winding only and do not links with the secondary winding. Similarly, some of the flux created by the secondary winding leaks through the air in the vicinity of the secondary winding only and do not links with the primary winding. This is known as the leakage flux and affects the performance of the transformer, ***but this is not a loss, as no energy has been lost in it.

7.23 **Applications of Transformers:** Some of the important applications of the transformers are given below for better understanding of the reader:

i) The most common use of the large power transformers and distribution transformers is in stepping up the voltages to high values for the bulk transmission of the electrical energy over long distances and then in stepping down to appropriate values for the distribution and use of electrical energy at a safe value.

ii) The transformers are being used in the voltage stabilizers for various industrial and domestic uses of electrical energy at a constant voltage.

iii) The small transformers are used as the voltage regulators in televisions, refrigerators, air-conditioners, computers and other appliances.

iv) The small transformers are used for stepping down the voltage to a low level for charging of mobile phones and toys *etc*.

v) The small transformers are used in radio transmitters and receivers, telephones, loud speakers etc.

vi) A special step-down transformer is used for obtaining a very high current for *electric arc welding*.

vii) A special step-down transformer is used in induction furnace for quick melting of the metals.

ix) A step-up transformer is used for the production of *X*-rays.

7.24 **Long Distance Transmission of Electrical Power:** The most important application of the transformers is for the stepping up the voltage level of power in the transmission of bulk electrical power over long distances and then for the stepping down the voltage level of power for its distribution to domestic consumers for its use at a safe potential. The bulk electrical power must be transmitted over long distance at very high potentials, because of the following drawbacks of low voltage transmission:

i) The resistance of a long transmission line is considerably high due to their large lengths (as $R \propto l$). Hence, a large amount of energy will be lost ($I^2 R\, t$) as heat in the transmission lines, if the voltage level of the electrical power is low (*consequently the current level of electrical power will be high*).

ii) A larger voltage drop ($I\,R$) will occur along the long transmission line, consequently the voltage at the receiving end will be much smaller than that of the sending end (generation station).

iii) The resistance of the transmission lines may be decreased by installing the thicker wires of large cross sectional area. But this will increase the installation cost and is not economical.

So, the long distance power transmission at low voltage and high current is neither efficient nor economical. If I is the current in transmission line and R is the equivalent resistance of the transmission line. The power wastage in the transmission line may be given as:

$$P_{Line\ Losses} = I^2 R \qquad (7.125)$$

The power delivered by the generator to the transmission line for bulk transmission of power may be given as:

$$P = V\,I \qquad \text{or,} \qquad I = \frac{P}{V} \qquad (7.126)$$

So, the current level (I) for a constant power (P) will be low if the voltage level (V) is high.

A design set up from the generation of electricity [at high voltage (HV A.C.)], through the long distance transmission of bulk electrical power [at Extra High Voltage (EHV A.C.)] to the end users (domestic users) is shown in the Fig. 7.88. The first transformer is shown at the location of the generation station at 11 kV or 33 kV. This HV A.C. is then step up to 220 kV (EHV A.C.) to deliver it to a pool of various generation stations connected through long transmission lines and collectively known as the ***National Grid***. The bulk transmission of electrical power is then done over long distances at 220 kV or 132 kV

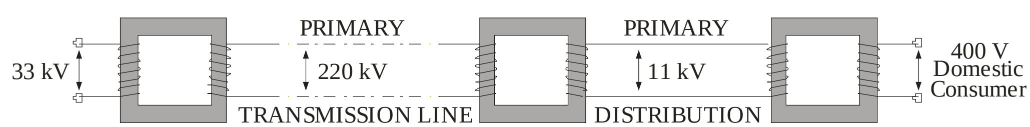


Fig. 7.88

(EHV A.C.) level. The second transformer is shown at the location of the outskirts of our cities to step down the voltage level to 33 kV or 11 kV for its primary distribution inside our cities. The third transformer is show at the location of a substation near by our homes to further step it down to domestic voltage level of 400 V for its secondary distribution up to our homes.

Problem 7.139: *The primary coil of an ideal step-up transformer has* 100 *turns and the transformation ratio is also* 100. *The input voltage and the power are* 220 *V and* 1100 *W respectively. Determine: i*) *the number of turns in the secondary winding, ii*) *the current through the primary winding, iii*) *the voltage across the secondary winding, iv*) *the current through the secondary winding, v*) *the power delivered by the secondary winding to the load.* [CBSE 2005-06]

Solution: $N_1 = 100$ Turns, $\frac{N_2}{N_1} = 100$, $V_1 = 220$ V, $P_1 = 1100$ W

The number of turns in the secondary winding may be given by the expression:

$$\frac{N_2}{N_1} = 100 \qquad \text{or,} \qquad N_2 = 100 \times N_1 = 100 \times 100 = 10000 \text{ Turns}$$

The current through the primary winding may be given as:

$$I_1 = \frac{P_1}{V_1} = \frac{1100}{220} = 5 \text{ A}$$

The voltage across the secondary winding may be given by the expression:

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} \qquad \text{or,} \qquad V_2 = \frac{N_2}{N_1} \times V_1 = 220 \times 100 = 22000 \text{ V} = 22 \text{ kV}$$

The current through the secondary winding may be given by the expression:

$$\frac{I_2}{I_1} = \frac{N_1}{N_2} \qquad \text{or,} \qquad I_2 = \frac{N_1}{N_2} \times I_1 = \frac{1}{100} \times 5 = 0.05 \text{ A} = 50 \text{ mA}$$

The power delivered by the secondary winding to the load may be given as:

$$P_2 = V_2 \times I_2 = 22000 \times 0.05 = 1100 \text{ W} = P_1$$

Problem 7.140: *Determine the current drawn by the primary winding of a transformer, which steps down a voltage of* 220 *V at primary side to a voltage of* 22 *V at secondary side to operate a device with an impedance of* 220 Ω. [Haryana 2000-01 CBSE 2007-08]

Solution: $V_1 = 220$ V, $V_2 = 22$ V, $Z = 220\ \Omega$

The current drawn by the impedance of 220 Ω at secondary side may be given as:

$$I_2 = \frac{V_2}{Z} = \frac{22}{220} = 0.1 \text{ A}$$

The corresponding current at primary side of the transformer may be given by the expression:

$$\frac{I_1}{I_2} = \frac{V_2}{V_1} \qquad \text{or,} \qquad I_1 = \frac{V_2}{V_1} \times I_2 = \frac{22}{220} \times 0.1 = 0.01 \text{ A} = 10 \text{ mA}$$

Problem 7.141: *A transformer has* 500 *turns in the primary winding and* 1000 *turns in the secondary winding. The primary voltage is* 200 *V and the load on the secondary side is* 100 Ω. *Determine the current on the primary side assuming it to be an ideal transformer.* [ISCE 2001-02]

Solution: $N_1 = 500$ Turns, $N_2 = 1000$ Turns, $V_1 = 200$ V, $R_L = 100\ \Omega$

The voltage at the secondary side may be given as:

$$V_2 = \frac{N_2}{N_1} \times V_1 = \frac{1000}{500} \times 200 = 400 \text{ V}$$

The current drawn by the load on secondary side may be given as:

$$I_2 = \frac{V_2}{R_L} = \frac{400}{100} = 4 \text{ A}$$

The corresponding current at primary side of the transformer may be given by the expression:

$$\frac{I_1}{I_2} = \frac{V_2}{V_1} \qquad \text{or,} \qquad I_1 = \frac{V_2}{V_1} \times I_2 = \frac{400}{200} \times 4 = 8 \text{ A}$$

Problem 7.142: *An ideal transformer has* 200 *primary and* 1000 *secondary turns. If the power input to the primary side is* 10 *kW at* 200 *V, determine: i*) *the output voltage, ii*) *the current drawn by the primary winding and the corresponding current at secondary side.* [CBSE 2001-02]

Solution: N_1 = 200 Turns, N_2 = 1000 Turns, P_1 = 10 kW, V_1 = 200 V

The output voltage at the secondary side may be given as:

$$V_2 = \frac{N_2}{N_1} \times V_1 = \frac{1000}{200} \times 200 = 1000 \text{ V} = 1 \text{ kV}$$

The current drawn by the primary winding and the corresponding current at secondary side may respectively be given as:

$$I_1 = \frac{P_1}{V_1} = \frac{10 \times 10^3}{200} = 50 \text{ A}$$

and, $$I_2 = \frac{N_1}{N_2} \times I_1 = \frac{200}{1000} \times 50 = 10 \text{ A}$$

Problem 7.143: *The output voltage of an ideal transformer, connected to a* 240 *V a.c. mains is* 24 *V. If this transformer is used to supply a lamp of ratings* 24 *V,* 24 *W; determine the current drawn by the primary side of the transformer from the a.c. mains.* [CBSE 1999-2000]

Solution: V_1 = 240 V, V_2 = 24 V, Lamp = 24 V, 24 W

The current dawn by the lamp connected at secondary side (24 V) of the transformer may be given as:

$$I_2 = \frac{P}{V_2} = \frac{24}{24} = 1 \text{ A}$$

The corresponding current at primary side of the transformer may be given by the expression:

$$\frac{I_1}{I_2} = \frac{V_2}{V_1} \qquad \text{or,} \qquad I_1 = \frac{V_2}{V_1} \times I_2 = \frac{24}{240} \times 1 = 0.1 \text{ A} = 100 \text{ mA}$$

Problem 7.144: *A* 100% *efficient transformer has* 200 *primary turns and* 40000 *secondary turns. The primary side of the transformer is connected to* 220 *V a.c. mains and the secondary side feeds a resistor of* 100 kΩ*. Determine the output potential difference per turn and the power delivered to the load on secondary side.*

Solution: η = 100%, N_1 = 200 Turns, N_2 = 40000 Turns, V_1 = 220 V, R_L = 100 kΩ

The output potential difference per turn may be given as:

$$\text{Output P.D.}_{per\ turn} = \frac{V_2}{N_2} = \frac{V_1}{N_1} = \frac{220}{200} = 1.1 \text{ V/Turn}$$

The output voltage at secondary side may be given as:

$$V_2 = \text{Output P.D.}_{per\ turn} \times N_2 = 1.1 \times 40000 = 44000 \text{ V} = 44 \text{ kV}$$

The power delivered to the load may be given as:

$$P_2 = \frac{V_2^2}{R_L} = \frac{(44000)^2}{100 \times 10^3} = 19.36 \text{ kW}$$

Problem 7.145: *A step down transformer is used to reduce the main supply from* 220 *V at primary side to* 11 *V at secondary side. If the current drawn by the primary winding is* 5 *A and the current delivered by the secondary winding to the load is* 95 *A, determine the efficiency of the transformer.*

Solution: $V_1 = 220$ V, $V_2 = 11$ V, $I_1 = 5$ A, $I_2 = 95$ A

The efficiency of a transformer may be given as:

$$\eta = \frac{\text{Power Output}}{\text{Power Input}} \times 100\% = \frac{V_2\, I_2}{V_1\, I_1} \times 100\% = \frac{11 \times 95}{220 \times 5} \times 100\% = 95\%$$

Problem 7.146: *Determine the current drawn by the primary winding of a transformer, which steps down the primary voltage of* 200 *V to secondary voltage at* 20 *V to operate a device having a resistance of* 20 Ω. *Assume the efficiency of the transformer to be* 80%.

Solution: $V_1 = 200$ V, $V_2 = 20$ V, $R_L = 20\ \Omega$, $\eta = 80\%$

The current drawn by the device connected to the secondary side of the transformer may be given as:

$$I_2 = \frac{V_2}{R_L} = \frac{20}{20} = 1 \text{ A}$$

The expression for the efficiency of a transformer may be given as:

$$\eta = \frac{\text{Power Output}}{\text{Power Input}} \times 100\% = \frac{V_2\, I_2}{V_1\, I_1} \times 100\% = 80\% = 0.8$$

So, $$I_1 = \frac{V_2\, I_2}{0.8 \times V_1} = \frac{20 \times 1}{0.8 \times 200} = 0.125 \text{ A}$$

Problem 7.147: *A* 10 *kW transformer, having* 20 *primary turns and* 100 *secondary turns, is connected across an a.c. supply of emf e* = 600 *sin* 314*t Volts. Determine: i) the maximum value of the flux in the transformer core, ii) the maximum value of the secondary voltage.*

Solution: $P = 10$ kW, $N_1 = 20$ Turns, $N_2 = 100$ Turns, $e = 600 \sin 314t$ Volts

Comparing the given emf expression with the standard emf expression, we will get:

$(E_1)_0 = 600$ V, $\omega = 314$ rad/sec

Let us assume that the flux in the transformer core is:

$$\phi = \phi_0 \cos \omega t$$

Since, the transformer is ideal with zero winding resistance and zero leakage reactance, the emf induced in the primary winding is equal to the applied voltage across the primary winding and may be given as:

$$e_1 = -N_1 \frac{d\phi}{dt} = -N_1 \frac{d}{dt}(\phi_0 \cos \omega t) = N_1\, \omega\, \phi_0 \sin \omega t = (E_1)_0 \sin \omega t$$

Where, $(E_1)_0 = N_1\, \omega\, \phi_0$

or, $$\phi_0 = \frac{(E_1)_0}{N_1\, \omega} = \frac{600}{20 \times 314} = 95.54 \text{ mWb}$$

The maximum value of the secondary voltage may be given as:

$$(E_2)_0 = \frac{N_2}{N_1} \times (E_1)_0 = \frac{100}{20} \times 600 = 3000 \text{ V} = 3 \text{ kV}$$

Problem 7.148: *A transformer, having* 400 *primary turns and* 2000 *secondary turns, is supplying a load of* 12.1 *kW at* 1100 *V connected to its secondary side. Determine the input voltage of the transformer at primary side.*

If the resistance of the primary winding is 0.2 Ω*, the resistance of the secondary side is* 2 Ω *and the transformer is* 90% *efficient, determine the heat losses in the two individual sides/windings* (*primary and secondary*) *of the transformer.*

Solution: N_1 = 400 Turns, N_2 = 2000 Turns, P_{output} = 12.1 kW, V_2 = 1100 V

R_1 = 0.2 Ω, R_2 = 2 Ω, η = 90%

The input voltage at the primary side may be given as:

$$V_1 = \frac{N_1}{N_2} \times V_2 = \frac{400}{2000} \times 1100 = 220 \text{ V}$$

The output current at the secondary side of the transformer may be given as:

$$I_2 = \frac{P_{output}}{V_2} = \frac{12.1 \times 10^3}{1100} = 11 \text{ A}$$

The expression for the efficiency of a transformer may be given as:

$$\eta = \frac{\text{Power Output}}{\text{Power Input}} \times 100\% = \frac{P_{output}}{V_1 I_1} \times 100\% = 90\% = 0.9$$

or, $$I_1 = \frac{P_{output}}{0.9 \times V_1} = \frac{12.1 \times 10^3}{0.9 \times 220} = 61.11 \text{ A}$$

So, the heat losses at the primary and secondary side may respectively be given as:

$$(P_{losses})_1 = I_1^2 \times R_1 = (61.11)^2 \times 0.2 = 746.886 \text{ W}$$

$$(P_{losses})_2 = I_2^2 \times R_2 = (11)^2 \times 2 = 242 \text{ W}$$

Problem 7.149: *A transformer, having* 300 *primary turns and* 2400 *secondary turns, is connected across a* 230 *V on its primary side. Determine the output voltage at the secondary side of the transformer.* [NCERT]

Solution: N_1 = 300 Turns, N_2 = 2400 Turns, V_1 = 230 V

The output voltage at the secondary side may be given as:

$$V_2 = \frac{N_2}{N_1} \times V_1 = \frac{2400}{300} \times 230 = 1840 \text{ V} = 1.84 \text{ kV}$$

Problem 7.150: *A transformer, having* 200 *primary turns and* 150 *secondary turns, is supplying a load at* 300 *V connected to its secondary side. Determine the input voltage at the primary side of the transformer.* [NCERT]

Solution: N_1 = 200 Turns, N_2 = 150 Turns, V_2 = 300 V

The input voltage at the primary side may be given as:

$$V_1 = \frac{N_1}{N_2} \times V_2 = \frac{200}{150} \times 300 = 400 \text{ V}$$

Problem 7.151: *The ratio of the number of turns in the primary and the secondary side of a step up transformer is* 1 : 200. *If it is connected to* 200 *V a.c. mains, determine the output voltage at the secondary side of the transformer. Also, determine the maximum value of the current in the secondary winding, if the transformer is drawing a primary current of* 2 *A.*

Solution: $N_1 : N_2 = 1 : 200$, $V_1 = 200$ V, $I_1 = 2$ A

The output voltage at the secondary side may be given as:

$$V_2 = \frac{N_2}{N_1} \times V_1 = \frac{200}{1} \times 200 = 40000 \text{ V} = 40 \text{ kV}$$

The output current (rms value) flowing through the secondary winding may be given as:

$$I_2 = \frac{N_1}{N_2} \times I_1 = \frac{1}{200} \times 2 = 0.01 \text{ A} = 10 \text{ mA}$$

The maximum value of the current in secondary winding may be given as:

$$(I_2)_0 = \sqrt{2} \times I_2 = \sqrt{2} \times 0.01 = 0.01414 \text{ A} = 14.14 \text{ mA}$$

Problem 7.152: *A* 100% *efficient transformer, having* 500 *primary turns and* 10000 *secondary turns, is connected across a* 220 *V a.c. mains on its primary side. Determine the output voltage at the secondary side of the transformer.*

Solution: $\eta = 100\%$, $N_1 = 500$ Turns, $N_2 = 10000$ Turns, $V_1 = 220$ V

The output voltage at the secondary side may be given as:

$$V_2 = \frac{N_2}{N_1} \times V_1 = \frac{10000}{500} \times 220 = 4400 \text{ V} = 4.4 \text{ kV}$$

Problem 7.153: *When a supply voltage of* 120 *V is impressed upon the primary side of a transformer, the current in the primary winding is found to be* 1.85 *A. If the efficiency of the transformer is* 95 % *and it is delivering a current of* 150 *mA at secondary side, determine the output voltage at the secondary side.*

Solution: $V_1 = 120$ V, $I_1 = 1.85$ A, $\eta = 95\%$, $I_2 = 150$ mA

The expression for the efficiency of a transformer may be given as:

$$\eta = \frac{\text{Power Output}}{\text{Power Input}} \times 100\% = \frac{V_2 I_2}{V_1 I_1} \times 100\% = 95\% = 0.95$$

or, $$V_2 = \frac{0.95 \times V_1 I_1}{I_2} = \frac{0.95 \times 120 \times 1.85}{150 \times 10^{-3}} = 1406 \text{ V} = 1.406 \text{ kV}$$

Problem 7.154: *A town situated* 20 *km away from a power plant generating power at* 440 *V, requires* 0.6 *MW of electric power at* 200 *V. The resistance of the two wire line carrying the current is* 0.04 *Ω/km. The town gets power from the line through a* 3000-220 *V step down transformer at a substation at the town. Determine: i) the line power losses in the form of heat, ii) the power supplied by the plant, assuming negligible power losses due to leakage.*

Solution: $l = 20$ km, $V_{generation} = 440$ V, $P_{recieving} = 0.6$ MW, $V_{recieving} = 200$ V,

$R_{line} = 0.04$ Ω/km, Line = 3000 – 220 V

The setup for generation (440 V) at sending end, step-up for transmission and transmission (3000 V), and step down (200 V) at receiving end is shown in the Fig. 7.89. The voltage at sending end is 440 V and that on the receiving end is 200 V. The transformer at receiving end has a transformation ratio of 3000V / 220 V.

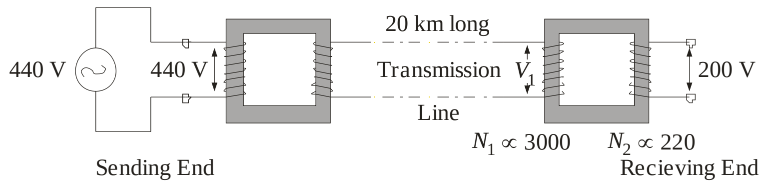


Fig. 7.89

The voltage of the transmission line may be given as:

$$V_1 = \frac{3000}{220} \times 200 = 2727.27 \text{ V}$$

The current corresponding to 0.6 MW power at 2727.27 V may be given as:

$$I_1 = \frac{P}{V_1} = \frac{0.6 \times 10^6}{2727.27} = 220 \text{ A}$$

The net resistance of the transmission line may be given as:

$$(R_{line})_{net} = 2 \times l \times R_{line} = 2 \times 20 \times 0.04 = 1.6\ \Omega$$

So, the power losses in the line as heat may be given as:

$$P_{losses} = I_1^2 \times R_1 = (220)^2 \times 1.6 = 77.44 \text{ kW}$$

And, the power supplied by the plant may be given as:

$$P_{sending} = P_{recieving} + P_{losses} = 600 + 77.44 = 677.44 \text{ kW}$$

Problem 7.155: *A transformer, having* 200 *primary turns and* 1000 *secondary turns, is supplying a load of* 9 *kW at* 1000 *V on its secondary side. Determine: i*) *the input voltage at primary side, ii*) *the heat loss in the primary winding, if the resistance of primary winding is* 0.2 Ω *and the efficiency of the transformer is* 90%*.*

Solution: $N_1 = 200$ Turns, $N_2 = 1000$ Turns, $P_{output} = 9$ kW, $V_2 = 1000$ V,

$R_1 = 0.2\ \Omega$, $\eta = 90\%$

The input voltage at the primary side may be given as:

$$V_1 = \frac{N_1}{N_2} \times V_2 = \frac{200}{1000} \times 1000 = 200 \text{ V}$$

The expression for the efficiency of a transformer may be given as:

$$\eta = \frac{\text{Power Output}}{\text{Power Input}} \times 100\% = \frac{P_{output}}{V_1 I_1} \times 100\% = 90\% = 0.9$$

or, $$I_1 = \frac{P_{output}}{0.9 \times V_1} = \frac{9 \times 10^3}{0.9 \times 200} = 50 \text{ A}$$

So, the heat losses at the primary side may be given as:

$$(P_{losses})_1 = I_1^2 \times R_1 = (50)^2 \times 0.2 = 500 \text{ W}$$

7.25 A.C. Generator: The present form of the generators is given to us by the great *Yugoslav Scientist*, *Nikola Tesla* in year 1988. The name generator used for the generation of electricity is actually a misnomer.

An a.c. generator / dynamo / synchronous generator is an electrical machine which converts mechanical energy into electrical energy.

An a.c. generator produces an alternating current that alternates its polarity (from positive to negative and vice-versa) regularly after a fixed interval of time, and the instantaneous value of the current changes continuously in a sinusoidal manner between it's positive maximum to negative maximum value.

Principle of Operation: If a coil is being rotated in a fixed magnetic field at a constant angular speed with its axis perpendicular to the magnetic field, the magnetic flux lines associated with the coil changes continuously (in sinusoidal manner) and an *EMF* is being induced in the coil according to *Faraday's Law of Electromagnetic Induction*, *i.e.* $e = -N\dfrac{d\phi}{dt}$.

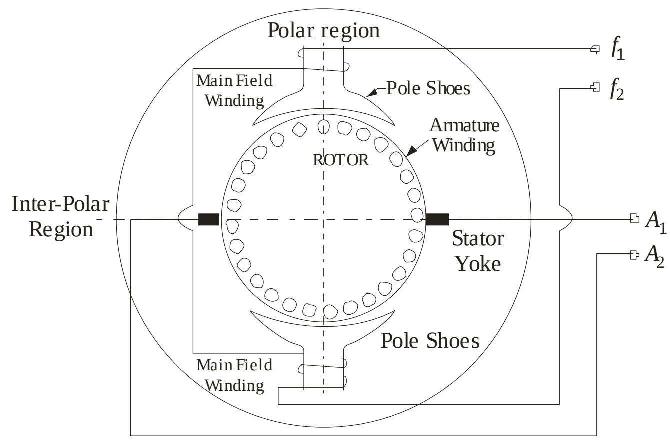
Construction of A.C. Generator: Every rotating electrical machine has two basic parts for the support of its components: *i*) *Stator* (stationary part), *ii*) *Rotor* (rotating part).

***i*) Stator:** The stator of an A.C. generator consists of following parts:

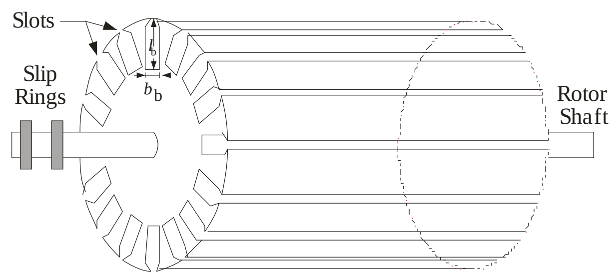
***a*) Yoke:** It is the soft iron structure to mount/support the components of stator inside it, [Fig. 7.90 (a)].

***b*) Pole Shoes:** They are made up of either permanent magnet in small dynamos or of laminated steel punching to support field winding to create a strong electromagnet in case of large generators. The laminated punching are used to reduce eddy current losses, [Fig. 7.90 (a)].

***c*) Carbon brushes assembly:** It carries two carbon brushes [one for (+)ve terminal and one for (–)ve terminal] to collect the A.C. current from rotating slip rings. The out-put terminals of generator are connected to stationary carbon brushes, on stator. These carbon brushes are in continuous contact with the rotating slip rings due to a spring arrangement behind the carbon brushes, [Fig. 7.90 (a)].



(a) Construction of D.C. Machine



(b) Rotor Body

Fig. 7.90

***d*) Bearing assembly:** Stator also has bearings and end brackets to support the rotating armature coils on the rotor.

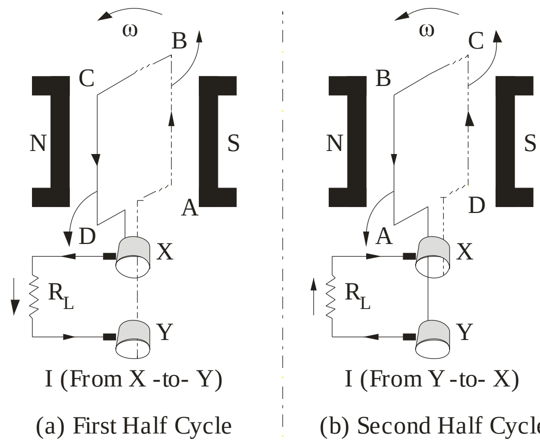
***ii*) Rotor:** Rotor is meant for rotating inside the stator assembly, it supports following components:

***a*) Rotor Body:** It is made up of laminated steel punching to reduce the eddy current losses. Rotor body has slots (open or closed) along its circumference to accommodate the armature windings inside them [Fig. 7.90 (b)].

***b*)** **Armature winding:** The insulated coils of copper wire accommodated in rotor slots (in which emf is being induced, when it rotates in the magnetic field with rotating rotor and hence can supply current to an electrical load) is known as armature winding. This armature winding is connected to the slip rings (on the rotor shaft) in order to supply the load current to an electrical load connected to the output terminals of the generator.

***c*)** **Slip Rings:** They are made up of H.T. steel and are located at rotor shaft, which gives continuous electrical contact of the carbon brushes over them to the ends of the rotating coil. They remains in continuous contact with stationary carbon brushes, located on stator to provide connection of armature winding to the stationary electrical load, [Fig. 7.90 (b)].

Working: Consider the setup shown in the Fig. 7.91, a coil *ABCD* is shown between two poles of a magnet (permanent magnet / electromagnet). The coil side *AB* is connected to the slip ring *Y*, and that the coil side *CD* is connected to another slip ring *X*. The coil sides will remain in the continuous contact with respective slip rings while rotating. The load resistance (R_L) is connected to the slip rings through the carbon brushes and spring arrangement, so that the rotating slip rings are in continuous contact with the stationary carbon brushes. Now, the coil is being rotated in anti-clock-wise direction between two poles of the magnet with the help of an external source (Prime Mover) giving mechanical energy to the coil. Consider the instant shown in Fig. 7.91 (a), when coil side *AB* is coming upwards and that the coil side *CD* is going downwards. The arrows are showing the direction of induced emf (and hence the current in the load) according to ***Flemings Right Hand Rule***, ***from X-to-Y, for this half rotation***. After half the rotation, the situation will become as shown in Fig. 7.91 (b), coil side *CD* is coming upwards and that the coil side *AB* is going downwards. The arrows are showing the direction of induced emf, for this half rotation (and hence the current in the load), ***from Y-to-X now***, ***which is opposite to the earlier direction of current***. It shows that the induced emf is alternating in nature for this rotating arrangement of the coil.



2 - Pole Single Phase A.C. (Synchronous) Generator

Fig. 7.91

Expression for Induced EMF in A.C. Generator: Let us define some parameters before deriving the emf equation for an A.C. Generator.

$N \rightarrow$ Number of turns in the armature winding.

$A \rightarrow$ Face area of each turn in armature winding.

$B \rightarrow$ Magnetic field (flux density).

$\theta \rightarrow$ Angle between the magnetic field and the normal to the plane of coil at any instant t.

$\omega \rightarrow$ Angular speed of the rotation of the armature coil.

We know that the flux linkages of a coil rotating at a constant angular speed changes in a sinusoidal manner. So, the flux linkages of the coil at any instant of the time may be given as:

$$\lambda = N\,\phi = N\,B\,A \cos\theta = N\,B\,A\cos\omega t \qquad (7.127)$$

The emf induced in the armature may be given, according to *Faraday's Law of Electromagnetic Induction*, as:

$$e = -\frac{d\lambda}{dt} = -\frac{d}{dt}(N\,B\,A\cos\omega t) = N\,B\,A\,\omega\sin\omega t = E_0 \sin\omega t \qquad (7.128)$$

Where, $E_0 = N\,B\,A\,\omega$ (7.129)

If a load of resistance R_L is connected across the armature terminals, the current through the load and armature winding may be given as:

$$I = \frac{e}{R_L} = \frac{E_0 \sin \omega t}{R_L} = I_0 \sin \omega t \tag{7.130}$$

So, $$I_0 = \frac{E_0}{R_L} = \frac{N\,B\,A\omega}{R_L} \tag{7.131}$$

So, both the induced emf as well as the current generated by the generator are varying in sinusoidal manner. The mechanical power is being supplied by the external agent (Prime Mover) to the armature coil, to which the armature coil is converting in the electrical energy to feed the electrical load to do some useful work.

Hydroelectric Power Generation Station: The water is stored in a dam up to a significant height. The water falls onto the giant turbines (water wheels) with a greater speed, under the influence of gravity, from the penstocks in the wall of the dam. These turbines are coupled with the rotor of heavy generators. The falling water from the penstocks rotate the turbines, which in turn rotate the rotor of heavy generator, to convert the potential energy of the water – through – its kinetic energy – to – turbine – into – the electrical energy by generator.

$$\begin{matrix}\text{Potential Energy}\\ \text{of Water Stored}\\ \text{at Height}\end{matrix} \rightarrow \begin{matrix}\text{Kinetic Energy}\\ \text{of Falling Water}\end{matrix} \rightarrow \begin{matrix}\text{Rotation of}\\ \text{Turbine}\end{matrix} \rightarrow \begin{matrix}\text{Rotation of}\\ \text{Armature Winding}\end{matrix} \rightarrow \text{Electrical Energy}$$

Thermal Power Generation Station: The superheated steam is produced in huge boilers using coal or crude oil in a thermal power station. The super heated steam emerging from small jets hit the turbines with a greater speed, due to the pressure of steam and orifice of jets. These turbines are coupled with the rotor of heavy generators. The steam hitting the turbines rotate the turbines which in turn rotate the rotor of heavy generator, to convert the potential energy of the steam – through – its kinetic energy – to – turbine – into – the electrical energy by generator.

$$\text{Heat} \rightarrow \begin{matrix}\text{Potential Energy}\\ \text{of Steam Stored}\\ \text{in Compressors}\end{matrix} \rightarrow \begin{matrix}\text{Kinetic Energy}\\ \text{of Steam}\end{matrix} \rightarrow \begin{matrix}\text{Rotation of}\\ \text{Turbine}\end{matrix} \rightarrow \begin{matrix}\text{Rotation of}\\ \text{Armature Winding}\end{matrix} \rightarrow \text{Electrical Energy}$$

Nuclear Power Generation Station: The superheated steam is produced in huge boilers using nuclear fuel in nuclear power stations, rest of the process of generating electrical power is same as that in a thermal power generation station.

7.26 Advantages and Dis-advantages of A.C. over D.C.: The alternating electric current has certain advantages and dis-advantages over the direct electric current, which are as enlisted below.

Advantages of A.C. over D.C.:

i) The generation of A.C. is more economical than D.C. because high voltage generation of D.C. is not possible (above 750 V), however the generation of A.C. is being done commercially at 11 kV, 33kV and 132 kV.

ii) The alternating current and voltages may very easily be stepped-up or stepped-down for the purpose of economical transmission and distribution.

iii) The alternating currents may easily be reduced by using choke coil without any significant wastage of electrical power.

iv) The alternating currents may easily be converted into direct currents with the help of rectifiers as and when required.

v) A.C. machines are rugged in construction. They are rough and tough and need less maintenance than that of the d.c. machines.

Dis-advantages of A.C. over D.C.:

i) Peak value of A.C. is higher ($I_0 = \sqrt{2}\, I_{rms}$) than that of the corresponding D.C. So, it is more dangerous to work with A.C. than that with the D.C.

ii) The direct current is required for the processes like: electroplating, electro-refining, electrotyping *etc*. These processes cannot be performed with A.C. supply.

iii) There is no skin effect in the direct currents (***Skin Effect*** *persists in alternating currents, which is the tendency of the current to concentrate on the surface of the conductor. So, more current flows at the surface of the conductor than that inside the conductor*). So, the required cross section of the conductor to carry the same d.c. current is smaller.

Problem 7.156: *A student pedals a stationary bicycle, the pedals of which are attached to a* 100 *turns coil of area* 0.1 m^2. *The coil rotates at half a revolution per second, when placed in a magnetic field of strength* 0.01 *Tesla perpendicular to the axis of rotation of the coil. Determine the maximum emf induced in the coil.* [NCERT, CBSE 2007-08]

Solution: N = 100 Turns, A = 0.1 m^2, n = 0.5 rps, B = 0.01 T

The maximum value of the emf induced in the coil may be given as:

$$E_0 = N\,B\,A\,\omega = N\,B\,A \times (2\,\pi\,f) = 100 \times 0.01 \times 0.1 \times (2\,\pi \times 0.5) = 0.314\text{ V}$$

Problem 7.157: *An a.c. generator, having a coil of* 50 *turns and area of* 2.5 m^2, *is rotating at an angular speed of* 60 *rad/sec in a uniform magnetic field of* 0.3 *Tesla between two fixed pole pieces. The resistance of the circuit including that of the coil is* 500 Ω. *Determine: i*) *maximum value of the current supplied by the generator, ii*) *the flux linkages of the coil when the current through the coil is zero, iii*) *would the generator works, if the coil is held stationary, while the pole pieces of the magnet would rotate at the same speed of rotation?*
[CBSE 1997-98, 2002-03]

Solution: N = 50 Turns, A = 2.5 m^2, ω = 60 rad/sec, B = 0.3 T, R = 500 Ω

The maximum value of the current supplied by the generator may be given as:

$$I_0 = \frac{E_0}{R} = \frac{N\,B\,A\omega}{R} = \frac{50 \times 0.3 \times 2.5 \times 60}{500} = 4.5\text{ A}$$

The alternating current generated in an a.c. generator may be given as: $i = N\,B\,A\,\omega \sin \omega t$

The value of this current will be maximum obviously for $\omega t = 0°$.

So, the flux linkages of the coil at this instant may be given as:

$$\lambda = N\,\phi = N\,B\,A \cos \omega t = 50 \times 0.3 \times 2.5 \times \cos 0° = 37.5\text{ Wb}$$

The generator will still work satisfactorily as there is a relative motion between the armature coil and the magnetic field, so the flux linkages of the coil are still changing at the same rate, and hence the same emf continues to be induced in the armature coil.

Problem 7.158: *An a.c. generator, having a coil of* 100 *turns and cross sectional area of* 3 m^2, *is rotating at a constant angular speed of* 60 *rad/sec in a uniform magnetic field of* 0.04 *T. The total resistance of the circuit is* 500 Ω. *Determine: i*) *the maximum current supplied by the current, ii*) *the power dissipation in the circuit.* [CBSE 2002-03]

Solution: N = 100 Turns, A = 3 m^2, ω = 60 rad/sec, B = 0.04 T, R = 500 Ω

The maximum value of the current supplied by the generator may be given as:

$$I_0 = \frac{E_0}{R} = \frac{N\,B\,A\omega}{R} = \frac{100\times 0.04\times 3\times 60}{500} = 1.44 \text{ A}$$

The power dissipated in the circuit may be given as:

$$P = I^2 \times R = \left(\frac{I_0}{\sqrt{2}}\right)^2 \times R = \left(\frac{1.44}{\sqrt{2}}\right)^2 \times 500 = 518.4 \text{ W}$$

Problem 7.159: *A generator develops an emf of* 120 *V and has a terminal potential difference of* 115 *V, when the armature current is* 25 *A. Determine the resistance of the armature.*

Solution: E = 120 V, V_t = 115 V, I_a = 25 A

The voltage relationship for the generator circuit may be given as:

$$E = V_t + I_a\,R_a$$

or, $$R_a = \frac{E - V_t}{I_a} = \frac{120-115}{25} = 0.2\ \Omega$$

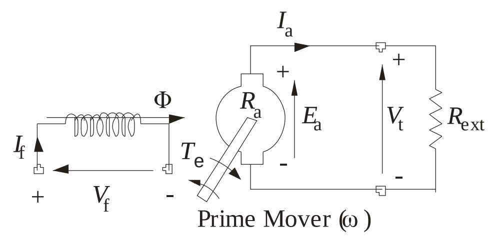


Fig. 7.92

Problem 7.160: *An armature coil, having* 20 *Turns of resistance* 15 Ω *and a cross sectional area of* 0.09 *m*^{2}*, is rotating at a constant frequency of* $\frac{150}{\pi}$ *Hz inside a uniform magnetic field of* 0.5 *Tesla. Determine the value of: i) maximum induced emf, ii) average induced emf.*

Solution: N = 20 Turns, R = 15 Ω, A = 0.09 m^2, $f = \frac{150}{\pi}$ Hz, B = 0.5 T

The maximum value of the induced emf in the armature coil may be given as:

$$E_0 = N\,B\,A\,\omega = N\,B\,A \times (2\,\pi\,f) = 20 \times 0.5 \times 0.09 \times 2\,\pi \times \frac{150}{\pi} = 270 \text{ V}$$

The average emf induced in the coil may be given as:

$$E_{avg} = \frac{2\,E_0}{\pi} = \frac{2\times 270}{\pi} = 171.89 \text{ V}$$

Problem 7.161: *An a.c. generator, having a coil of* 2000 *turns and a cross sectional area of* 80 *cm*^{2}*, is rotating at an angular speed of* 200 *rpm inside a uniform magnetic field of* 0.048 *Tesla. Determine the peak value and the rms value of the induced emf in the armature coil.* [Punjab 2001-02]

Solution: N = 2000 Turns, A = 80 cm^2, $f = 200 \text{ rpm} = \frac{200}{60} = \frac{10}{3}$ Hz, B = 0.048 T

The peak value of the emf induced in the armature coil may be given as:

$$E_0 = N\,B\,A\,\omega = N\,B\,A \times (2\,\pi \times f) = 2000 \times 0.048 \times 80 \times 10^{-4} \times 2\,\pi \times \frac{10}{3} = 16.085 \text{ V}$$

The rms value of the emf induced in the armature coil may be given as:

$$E_{rms} = \frac{E_0}{\sqrt{2}} = \frac{16.085}{\sqrt{2}} = 11.374 \text{ V}$$

SHORT ANSWER TYPE QUESTIONS FOR EXERCISE

1. Define following in context of alternating current: *i*) *alternating current*, *ii*) *waveform*, *iii*) *instantaneous value, iii*) *amplitude, iv*) *cycle, v*) *time period, vi*) *frequency, vii*) *angular frequency, vii*) *relationship between angular speed and frequency.*

2. Define and give the value following for sinusoidal alternating current: *i*) *average value*, *ii*) *rms / effective value, iii*) *standard equation for the alternating current.*

3. Define following in context of alternating current: *i*) *phasor diagram*, *ii*) *phase, iii*) *phase angle, iv*) *phase difference.*

4. Give the expression for the power dissipated and the value of phase difference between the applied voltage and the circuit current (lagging / leading) in case of: *i*) *purely resistive circuit*, *ii*) *purely inductive circuit*, *iii*) *purely capacitive circuit.*

5. Define and give the expression for the *time constant* of a *purely capacitive circuit* in case of a d.c. supply applied across the capacitor.

6. Draw and properly label (naming of branches of the triangles) the impedance triangles and power triangles for the following circuits: *i*) *R-L series circuit*, *ii*) *R-C series circuit*, *iii*) *R-L-C series circuit.* Also mention the unit of each power in power triangle.

7. Define: *i*) *power factor, ii*) *watt-full current*, *iii*) *watt-less current*; with the help of a suitable *phasor diagram*.

8. Define and give the condition for the *resonance* to occur in an *R-L-C series circuit*. Also, give the expression for: *i*) *resonance frequency, ii*) *impedance at resonance*, *iii*) *current at resonance, iv*) *power factor at resonance, v*) *quality factor* (*all three formulae*), *vi*) *band width, vii*) *half power frequencies.*

9. Draw the *current –vs– frequency curve* for an *R-L-C series circuit* and derive the expression for the *band width* and *half power frequencies*. Also, write down the physical significance of *half power frequencies* and the reason due to which the *R-L-C series circuit* is also known as a *tuner circuit*.

10. For an *R-L-C series circuit*, define the following: *i*) *quality factor*, *ii*) *selectivity*, *iii*) *relationship between quality factor and selectivity with the help of a suitable diagram*.

11. Derive the expressions for the energy associated with a: *i*) *pure inductor, ii*) *pure capacitor.*

12. Show that the *energy remains conserved* in case of an *L-C oscillator.*

13. Explain the *working principal* of a *transformer*, and name the parameters that will get transformed from *primary winding* to *secondary winding*. Also, derive the *emf equation* for a *transformer* and hence prove the relationship: $\frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{I_1}{I_2}$

14. Name and explain all the losses in a transformer and their cause. Also mention which loss is *variable loss* and which one is a *constant loss*.

15. Give the *applications of the transformers*. Also, explain: what will happen to a transformer if a d.c. supply is given to it?

16. Give the advantages and dis-advantages of A.C. supply over D.C. supply.

17. Explain the *working principal* of an *A.C. generator* with the help of suitable diagrams.